JAN 29

**Next Meeting January 29, 1904. Subject: Alternating-Current Railway Motors.**

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# TRANSACTIONS

OF THE

# AMERICAN INSTITUTE

OF

# ELECTRICAL ENGINEERS.

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V. 19

## AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

NEW YORK, October 23, 1903.

The 180th Meeting was held this date at Carnegie Hall, 154 West Fifty-seventh Street.

SECRETARY POPE:—At a meeting of the Board of Directors this afternoon the following Associates were elected:

| | | |
|---|---|---|
| ADAMS, ROY HARMON, | Assistant Electrical Engineer, Signal Corps, U. S. Army; res., 1823 Jackson Ave., New Orleans, La. | Townsend Wolcott. Edgar Russel. LeRoy Clark. |
| ANDERSON, PAUL LEWIS, | Tester, Sprague Electric Co. Bloomfield; res., 253 Renshaw Ave., E. Orange, N. J. | W. S. Franklin. Robert Lundell. J. H. Klinck. |
| BALDWIN, CHARLES FOWLER, | Chief Engineer Bell Telephone Mfg. Co., 18 Rue Bondewyns, Antwerp, Belgium. | F. R. Welles. G. A. Hamilton. E. O. Zwietusch. |
| BARTON, EDWARD GUSTAVUS CAMPBELL, | Managing Director, Brisbane Electric Supply Co., Ltd., Ann St., Brisbane, Queensland. | John Hesketh. A. J. Hanson. J. S. Fitzmaurice. |
| BRAM, ELMER E., | Superintendent and Electrical Engineer, Ashtabula Electric Light and Power Plant, Ashtabula, Ohio. | E. E. Gilbert. H. W. Hillman. J. L. Merrill. |
| BEAUCHAMP, LEON, | Manager and Electrical Engineer, Standard Construction Co., Montreal, Que. | L. A. Herdt. P. G. Gossler. Wm. Maver, Jr. |
| BENNETT, RALPH, | Chief Draftsman, Edison Electric Co., Los Angeles, Cal. | C. A. Copeland. E. B. Raymond. R. W. Pope. |
| BROOKE, JAMES DILLON, | Engineering Department of New York Telephone Co.; res., 509 Amsterdam Ave., New York City. | P. L. Griffith. J. J. Carty. Calvin W. Rice. |
| BYINGTON, ALBERT JACKSON, | Superintendent, Santos St. Railway Co., Santos Brazil, S. A. | M. A. Oudin. F. O. Blackwell. R. W. Pope. |
| CHACE, WILLIAM GREGORY, | Electrical Engineer, Mitchell & Jackson, Niagara Falls, res., St. Catharines, Ont. | T. R. Rosebrugh. L. B. Chubbuck. J. Sangster. |

| | | |
|---|---|---|
| CHAMBERS, FRANK ROSS, Jr., Draftsman, with E. W. Cummings, 70 Dexter Horton Building, Seattle, Wash. | A. E. Ransom.<br>M. P. Randolph.<br>A. A. Miller. |
| COLE, GEORGE PERCY, Motor Designing Engineer, Wagner Electric Mfg. Co.; res., 2712 Locust St., St. Louis, Mo. | W. A. Layman.<br>A. H. Timmerman.<br>R. W. Pope. |
| DAVIES, JOHN HUBERT, Senior Partner, Hubert Davies & Spain, Johannesburg, S. A. | Ch. Gorissen.<br>A. W. K. Peirce.<br>R. W. Pope. |
| FOOTE, FERDINAND JOHN, Erecting Engineer, Bullock Elec. Mfg. Co.; res., 4615 Montgomery Ave., Norwood, Ohio. | D. C. Jackson.<br>B. A. Behrend.<br>H. H. Cutler. |
| FUCHS, HUGO, Draftsman, Chicago Edison Co., 139 Adams St.; res., 317 Bissell St., Chicago, Ill. | H. M. Hope.<br>A. A. Radtke.<br>E. N. Lake. |
| FUCHS, GEORGE ADAM, Electrical Engineer, Electric Equipment Co., Erie, Pa. | C. B. Coates.<br>R. W. Pope.<br>W. D. Brown. |
| HASTINGS, LOUIS BROUNELL, Erecting Road Engineer, Stanley Electric Mfg. Co., Pittsfield, Mass. | C. C. Chesney.<br>D. B. Rushmore.<br>E. A. Bessey. |
| HILL, CLARENCE FREER, Electrical Engineer, Western Electric Co., Market St., Philadelphia, Pa. | H. P. Suman.<br>J. H. Finney.<br>Philander Betts. |
| HOFFMAN, WILLIAM LEVI, Electrical Engineer, Dynamo Test Department, Westinghouse Electric & Mfg. Co., Pittsburg; res., 834 Ramsey St., Wilkinsburg, Pa. | E. M. Olin.<br>H. J. Ryan.<br>R. W. Pope. |
| HUSTON, SAMUEL, Superintendent of Lines, Cauvery Power Scheme, Kaukanhalli, Mysore State, India. | F. O. Blackwell.<br>J. J. Mahony.<br>R. W. Pope. |
| JUHLIN, GUSTAV ADOLF, Assistant Electrical Engineer, Dick Kerr & Co., Ltd., West Strand Road, Preston, Eng. | A. P. Zani.<br>J. N. Dodd.<br>A. J. J. Pfeiffer. |
| KIRKER, GAYLORD BRENNAN, Erecting Engineer, Société Anonyme Westinghouse; res., 2 Boulevard Sadi Carnot, Lé Havre, France. | A. Schmid.<br>C. F. Scott.<br>R. D. Mershon. |
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| LOVERIDGE, IRVING, Manager, Bell Telephone Mfg. Co., 18 Rue Bondewyns, Antwerp, Belgium. | F. R. Welles.<br>G. A. Hamilton.<br>E. O. Zwietusch. |
| NEALL, NEWITT JACKSON, Electrical Engineer, Westinghouse Electric & Mfg. Co., Pittsburg; res., Edgewood Park, Pa. | P. H. Thomas.<br>C. F. Scott.<br>C. E. Skinner. |

| | |
|---|---|
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| PARKS, COLEMAN CLYDE, Draughtsman, Electrical Dept., Homestead Works of Carnegie Steel Co., Munhall, Pa. | W. G. Davis.<br>S. E. Johannesen.<br>H. P. Daniels. |
| PRINCE, FREDERICK WELLES, Superintendent of Construction, Hartford Electric Light Co.; res., 821 Broad St., Hartford, Conn. | H. J. Blakeslee.<br>W. L. Robb.<br>Francis Hodgkinson |
| RIKO, YASUO, Electrical Engineer, Department of Communications, Tokio, Japan. | G. A. Harris.<br>F. B. H. Paine.<br>C. I. Young. |
| SIBLEY, ROBERT, Assistant Electrical Engineering Department, University of California, Mechanics Building, Berkeley, Cal. | C. L. Cory.<br>J. N. LeConte.<br>A. M. Hunt. |
| SLOANE, THOMAS O'CONNOR, JR., Assistant in Electrical Engineering, Columbia University, New York City; res., 55 Montrose Ave., South Orange, N. J. | G. F. Sever.<br>F. B. Crocker.<br>Martin Arendt. |
| TEMPLE, WORRALL E. S., Constructing Engineer, Pocahontas Collieries Co., Pocahontas, Va. | E. E. Boyer.<br>Richard Fleming.<br>A. W. Schramm. |
| TRILLE, RADLEY MOTT, Electrical Engineer, United Lumber Co., Cienfuegos, Cuba. | Wm. Maver, Jr.<br>Townsend Wolcott.<br>H. A. Lardner. |
| WRIGHT, FRANK THOMAS, Draughtsman, Underground Electric Railways Co., of London, Ltd.; Hamilton House, Victoria Embankment; res., 27 Earls Court Gardens, Kensington, London, Eng. | S. B. Fortenbaugh.<br>E. R. Hill.<br>H. E. Molé. |

Total, 34.

## TRANSFERRED FROM ASSOCIATE TO THE GRADE OF MEMBER.

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WALTER ATWOOD HALL, Designing Engineer, General Electric Co., Lynn, Mass.

WILLIAM WEEDON COLE, President and General Manager, Elmira Water, Light, and Railway Co., Elmira, N. Y.

WALTER I. SLICHTER, Electrical Engineer, General Electric Co., Schenectady, N. Y.

*A paper presented at the 180th Meeting of the American Institute of Electrical Engineers, New York, Oct. 23, 1903.*
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# THE CONDITIONS GOVERNING THE RISE OF TEMPERATURE OF ELECTRIC RAILWAY MOTORS IN SERVICE.

BY CARY T. HUTCHINSON.

In a former paper read before the INSTITUTE (TRANS. AM. INST. E. E., Vol. XIX., p. 129), I described a method for determining the motor-capacity and energy required to make any schedule speed for a run of any distance, with electric motors. Since that time I have extended the method to include the predetermination of the rise of temperature of the motor when those constants of the motor are known, upon which the rise of temperature depends.

In that investigation the time required for each of the three

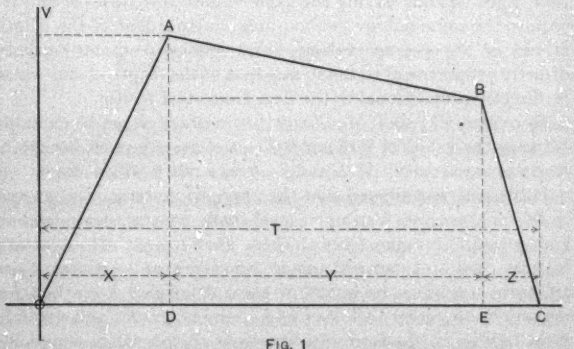



FIG. 1

periods of the " type-run." shown in Fig. 1, was first determined; this was an algebraical problem, the solution of which involved

the initial acceleration ($a$), the coasting retardation ($b$), the retardation after braking ($c$), the length of the run and the average speed from start to stop. These equations are:

$$x/T = 2A_0/a - K_1 (A_0 - A)^{\frac{1}{2}}$$
$$y/T = K_2 (A_0 - A)^{\frac{1}{2}}$$
$$z/T = 2A_0/c - K_3 (A_0 - A)^{\frac{1}{2}}$$

in which $K_1$, $K_2$, $K_3$ and $A_0$ are simple functions of ($a$), ($b$) and ($c$) only; $x$, $y$ and $z$ are respectively the times of initial acceleration, coasting and braking, and $T$ the total time, as shown in Fig. 1. The only quantity occurring in these equations other than the three accelerations, ($a$), ($b$) and ($c$), then, is

$$A = V/T = 0.682\, L/T^2$$

$T$ being the time and $V$ the average speed from start to stop; the form of the equation shows that all runs having the same values of ($a$), ($b$), ($c$) and $A$ will be made in the same proportionate time for the different periods, *i.e.*, the same fraction of the total time will be required for accelerating, coasting and braking. In other words, all runs having the same value of the accelerations and of $A = V/T$ are represented by the same figure, with merely a change in scale for the different lengths of run, the ordinates and abscissas being proportional to $L^{\frac{1}{2}}$.

This quantity, $A$, reduces the variables from three—$V$, $L$ and $T$, to two—$A$ and $L$. $A$ is a convenient quantity to use as independent variable for the curve-sheets; it simplifies the presentation, since all runs having the same values of $A$, ($a$), ($b$) and ($c$) require the same energy per ton-mile, independent of the length of run or the average velocity, and require a motor-capacity directly proportional to the square root of the length of run—that is, directly proportional to the time from start to stop.

Moreover, all values of $A$ occurring in practice can be included between the values of 0.15 and 0.35—a relatively small variation whereas variations in $V$, $L$ and $T$ cover a much wider range.

It is to be remembered that the previous investigation, as well as this one, assumes a straight, level track, constant train friction and constant braking force. These assumptions are necessary to any general treatment, since curvature and gradients are arbitrary. I have, however, in part, developed a method for making the necessary allowances for curves, grades and variable train friction in the form of corrections to the values found for the corresponding simple case.

These equations involve four quantities, of which two—($b$) and ($c$)—are assumed to be constants and two—($a$) and $A$—

the variables. The solution of the equations is best given by curve-sheets, with $x/T$, $y/T$ and $z/T$ as ordinates, and $A$ as abscissas; a separate curve is required for each value of $(a)$. CS 1 is plotted with

$(b) = .15$ mph/sec. $= 13.6$ lb. per ton

and $(c) = 2.0$ mph/sec. $= 182$ lb. per ton;

$(c)$ is the total retardation after braking, including train friction. This curve-sheet is similar to CS 2 of the first paper, save that there the values, $(b) = .2$ and $(c) = 3.0$ mph/sec., were used.

Fig. 1 shows the type case, with three constant accelerations;

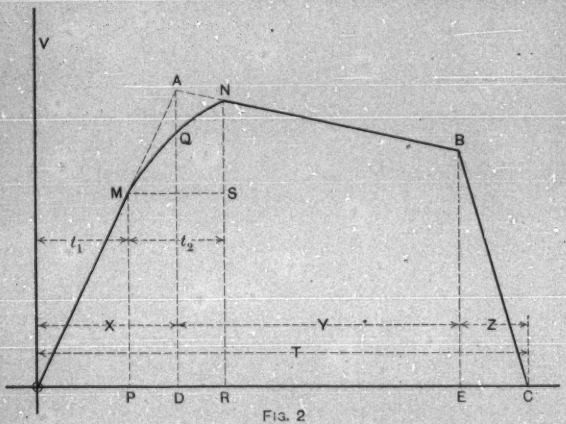



FIG. 2

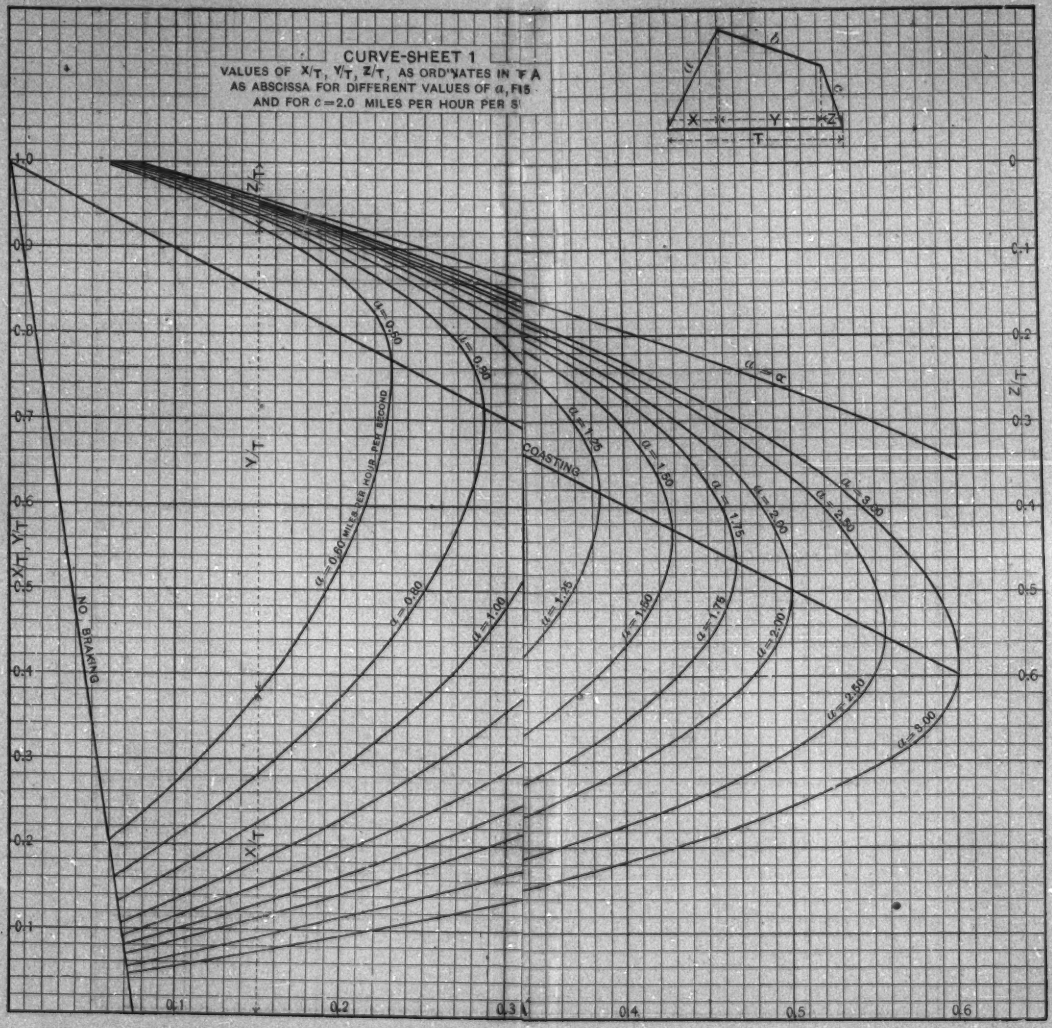
Fig. 2 the actual case, in which acceleration on the motor-curve is added to the constant initial acceleration; Fig. 2 then involves the characteristics of the motor used. CS 5 of the first paper gave the average speed- and torque-curves of a number of tramway motors in common use; I have since added other motor-curves, and in CS 2 show average motor-curves, which represent with practical accuracy the torque and speed relations of any General Electric or Westinghouse motor, throughout the range of ordinary operation. The extreme difference between the speed of any motor of either company and the speed on this curve-sheet for a given torque is greatest at the high speeds, but within the range of ordinary operation this variation is less than 5 per cent

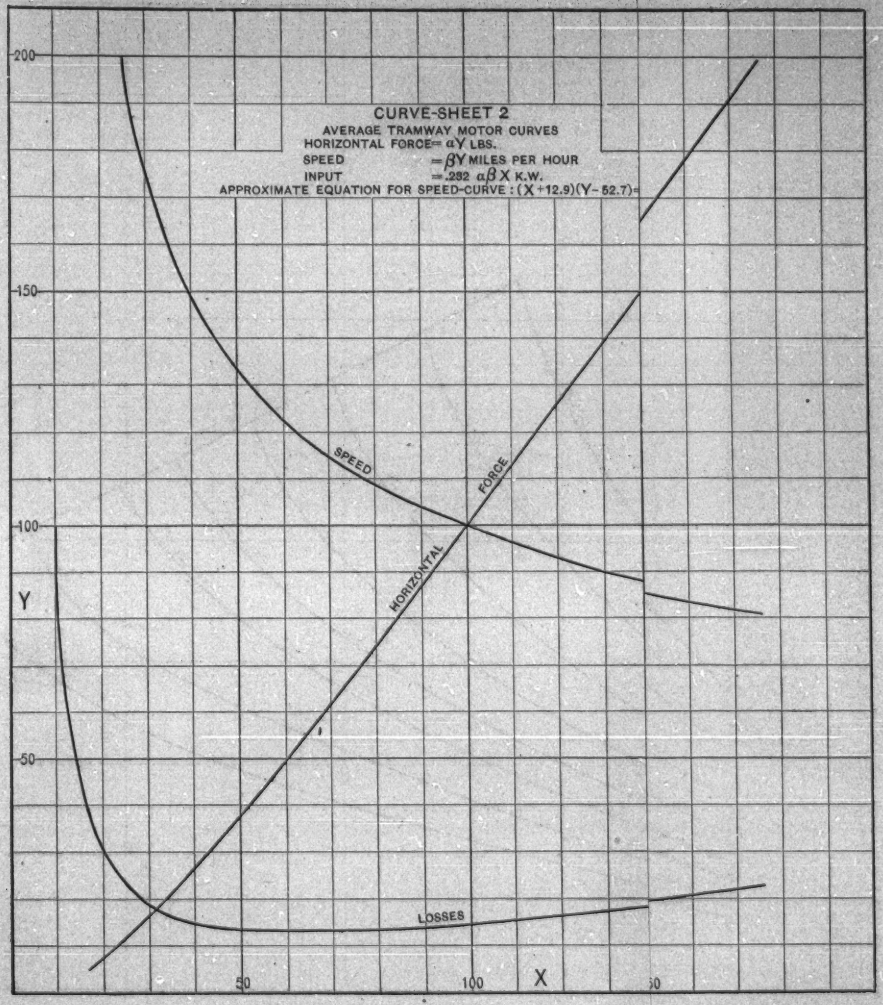
It is necessary to assume the same motor performance in order that the variation in energy consumption and motor-capacity required by the different schedules, when made with various initial and motor accelerations, may be shown independently of variations in characteristics of the motor. If one assumes other motor-curves, the methods outlined here can be applied by making certain corrections; but, as a matter of fact, the small variations in the shape of the motor-curve are of comparatively slight consequence in the final results.

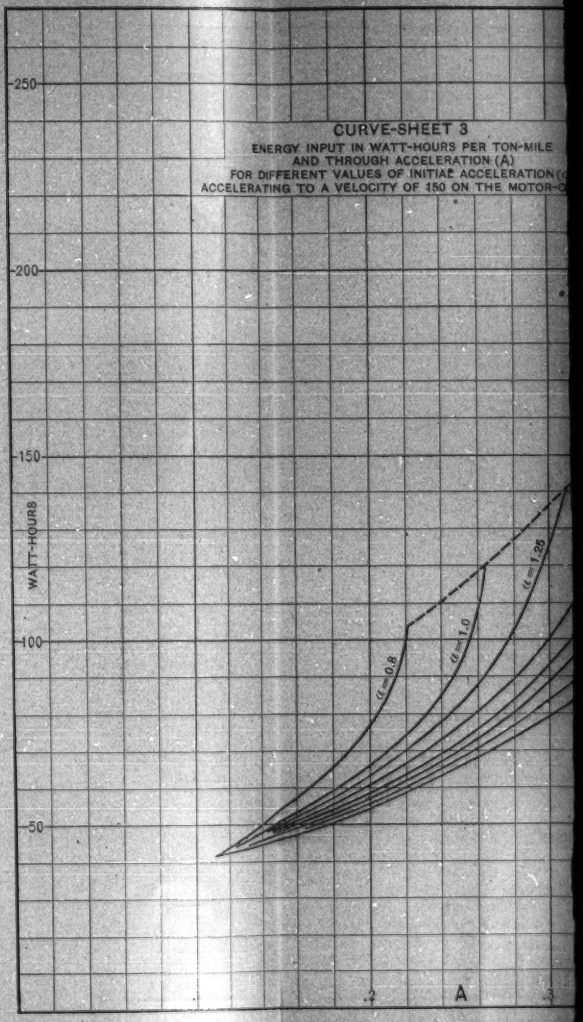
The efficiency of different motors varies through a much wider range than do the speed and torque; but the efficiency is used only in connecting the input and output, and not in determining heat losses, which are the subject of a separate consideration; hence considerable differences in efficiency curves are of minor consequence.

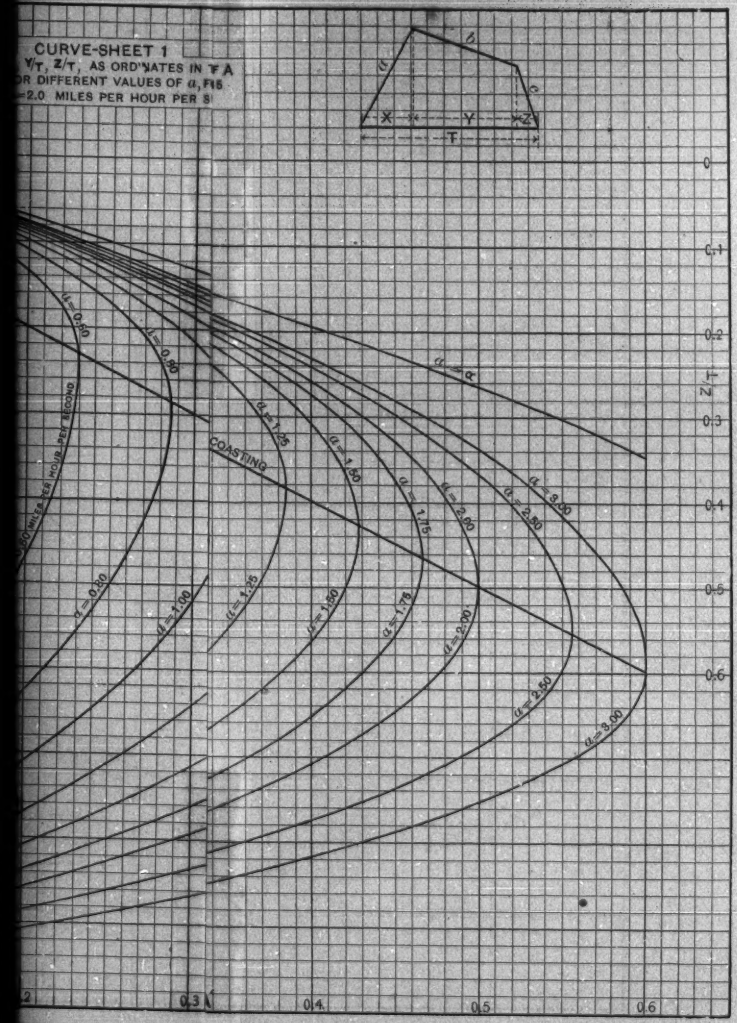
Here, as in the first paper, the datum point for these motor-curves is the load determined by the "hour-rating",—that is, the load the motor will carry for one hour with a temperature rise of 75° C. when tested on the stand, with covers off; this value is assumed to be the commutation-limit of the motor-capacity and all the results of the investigation are expressed in terms of the input donated by the datum point; therefore it only remains to determine the conditions of a run which will give the limiting temperature elevation when the motor is used at this particular input. The hour-rating for the maximum input is not unduly conservative; it leaves a margin for greater power on curves, grades and for emergencies.

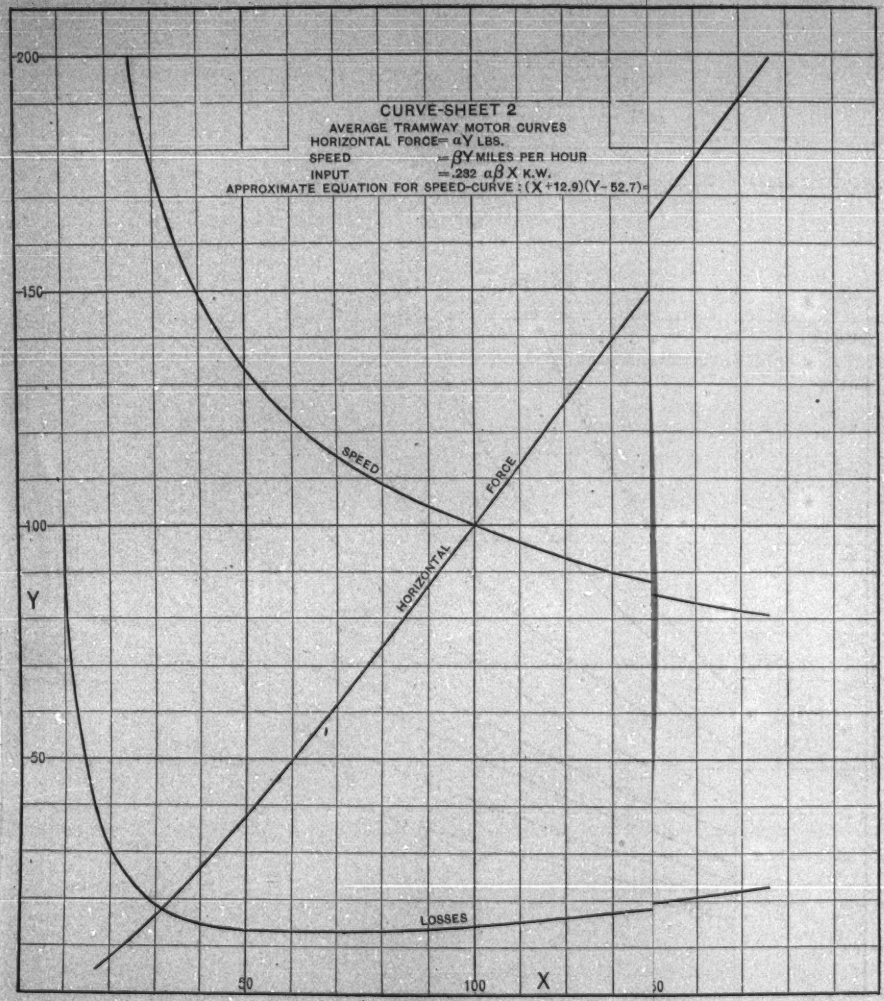
CS 3 and CS 4 give energy and motor-capacity in terms of through-acceleration $A$; they are the same as CS 13 and CS 14 of the first paper, except that here the abscissas are through acceleration with the motor-curve instead of the corresponding through-acceleration with the type-curve; that is, the quantity $A_m$ of the first paper is used as abscissa instead of the $A$ of that paper, but is designated $A$; and that the motor-capacity is given for a run of one mile. This change simplifies the use of the curves, in that it makes it unnecessary to add corrections to the value of $A$ calculated from the data given for the run. CS 3 then gives directly the energy per ton-mile for any run and for initial accelerations from 0.6 to 3.0 mph/sec., and for a train friction of 13.6 lbs. per ton, equivalent to $(b) = .15$, a retardation after braking equivalent to 2 mph/sec., and all for an acceleration on the motor-curve to a speed 50 per cent. greater than the speed with all resistance out.

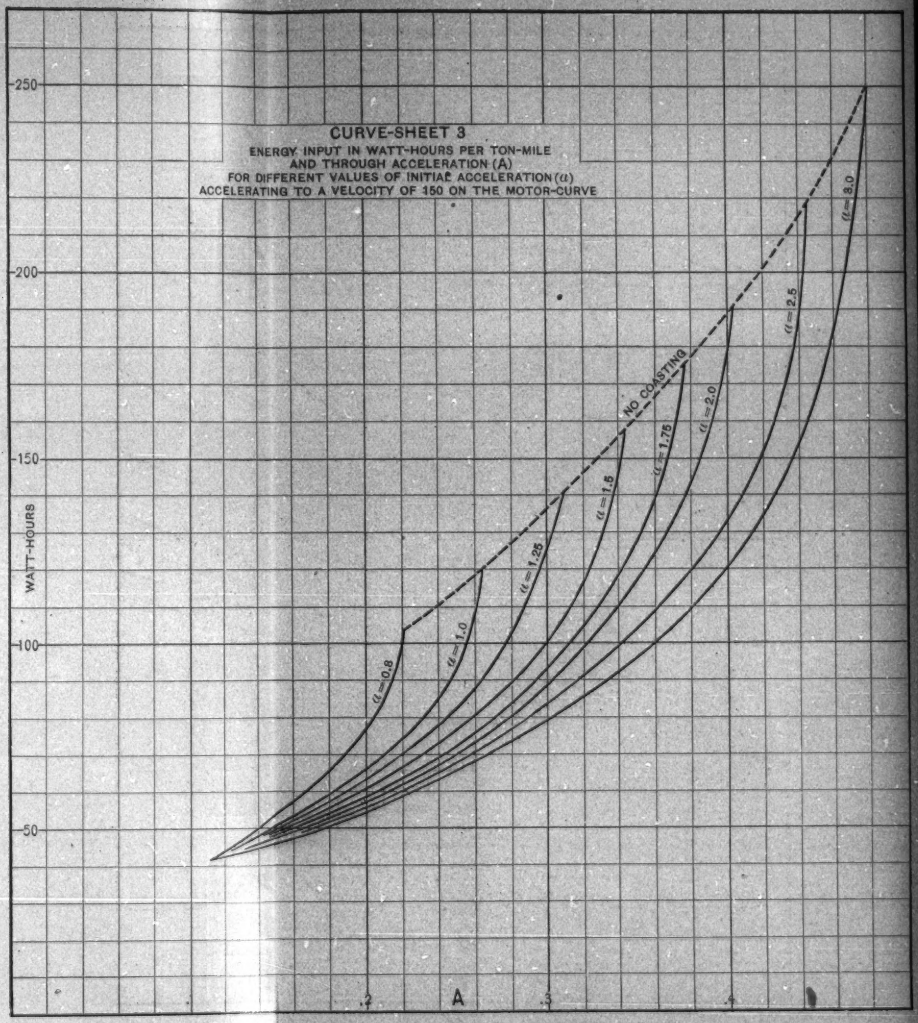

CURVE-SHEET 1
VALUES OF X/T, Y/T, Z/T, AS ORDINATES IN TERMS OF A
AS ABSCISSA FOR DIFFERENT VALUES OF α, FIS
AND FOR c=2.0 MILES PER HOUR PER S
A
B
C
X
Y
Z
T
Z/T
Y/T
X/T
α=0.50
α=0.80
α=1.00
α=1.25
α=1.50
α=1.75
α=2.00
α=2.50
α=3.00
α=α
COASTING
NO BRAKING
α=0.80 MILES PER HOUR PER SECOND
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

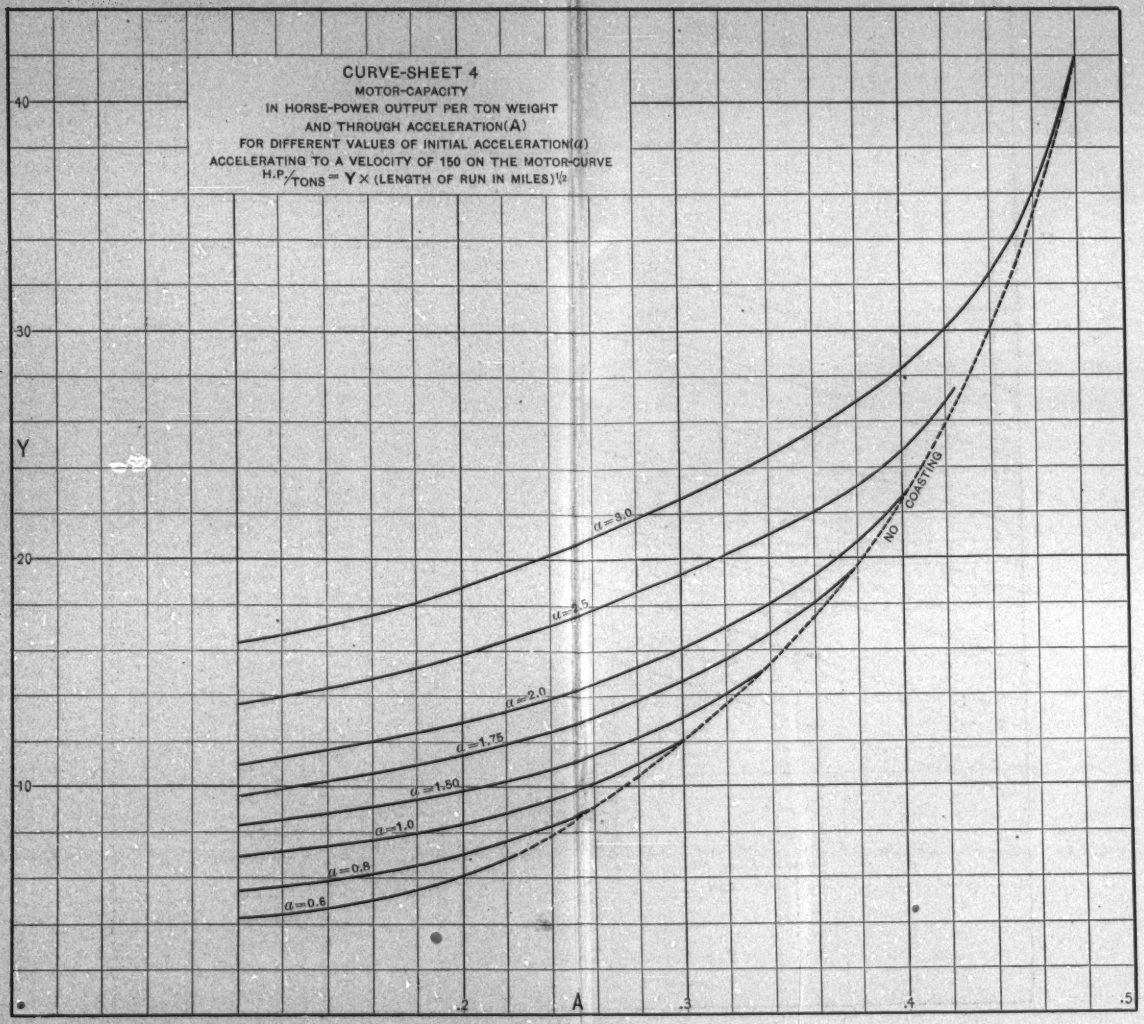

CURVE-SHEET 2
AVERAGE TRAMWAY MOTOR CURVES
HORIZONTAL FORCE = αY LBS.
SPEED = βY MILES PER HOUR
INPUT = .232 αβX K.W.
APPROXIMATE EQUATION FOR SPEED-CURVE : (X+12.9)(Y-52.7)=
200
150
100
50
Y
SPEED
FORCE
HORIZONTAL
LOSSES
50
100
X

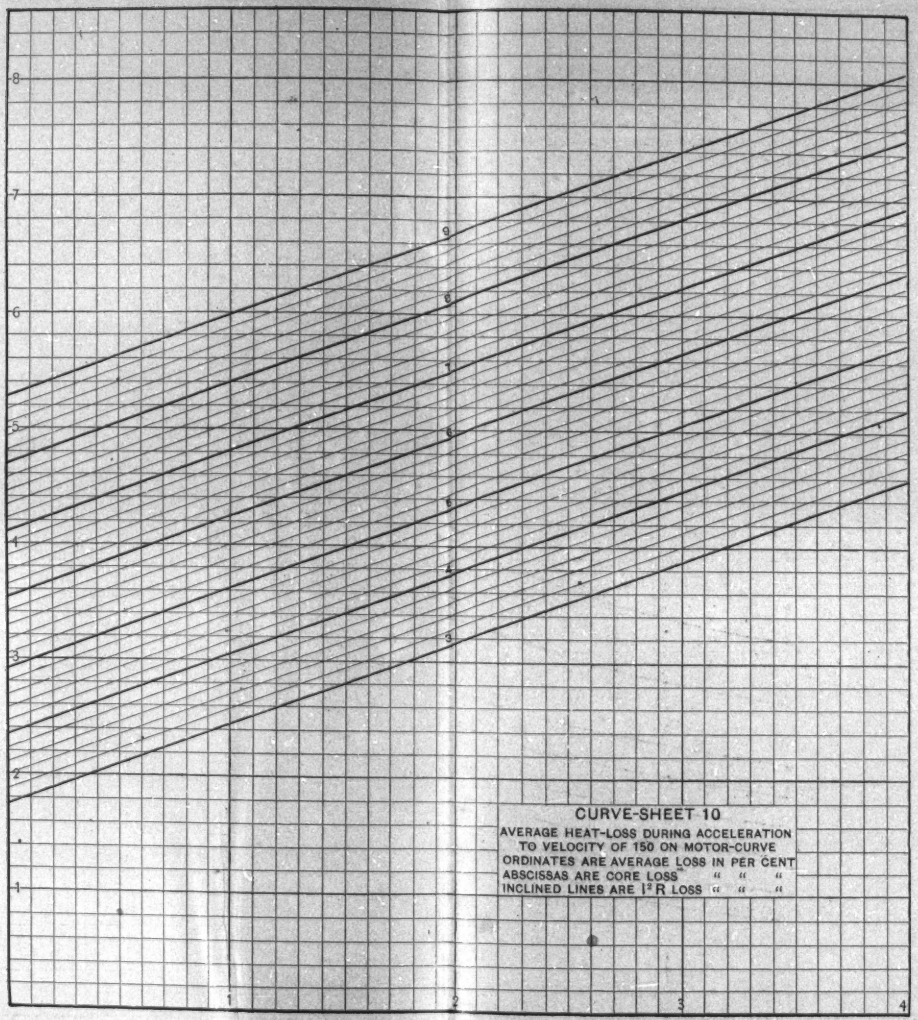

CURVE-SHEET 3
ENERGY INPUT IN WATT-HOURS PER TON-MILE
AND THROUGH ACCELERATION (A)
FOR DIFFERENT VALUES OF INITIAL ACCELERATION (α
ACCELERATING TO A VELOCITY OF 150 ON THE MOTOR-C
250
200
150
100
50
WATT-HOURS
α=0.8
α=1.0
α=1.25
A


CURVE-SHEET 1
Y/T, Z/T, AS ORD'NATES IN T A
OR DIFFERENT VALUES OF α, F15
=2.0 MILES PER HOUR PER S
X
Y
T
0
0.1
0.2
0.3
0.4
0.5
0.6
Z/T
COASTING
α = 0.50
α = 0.75
α = 1.00
α = 1.25
α = 1.50
α = 1.75
α = 2.00
α = 2.50
α = 3.00
α = α


CURVE-SHEET 2
AVERAGE TRAMWAY MOTOR CURVES
HORIZONTAL FORCE = αY LBS.
SPEED = βY MILES PER HOUR
INPUT = .232 αβX K.W.
APPROXIMATE EQUATION FOR SPEED-CURVE : (X+12.9)(Y-52.7)=
200
150
100
50
Y
SPEED
FORCE
HORIZONTAL
LOSSES
50
100
X

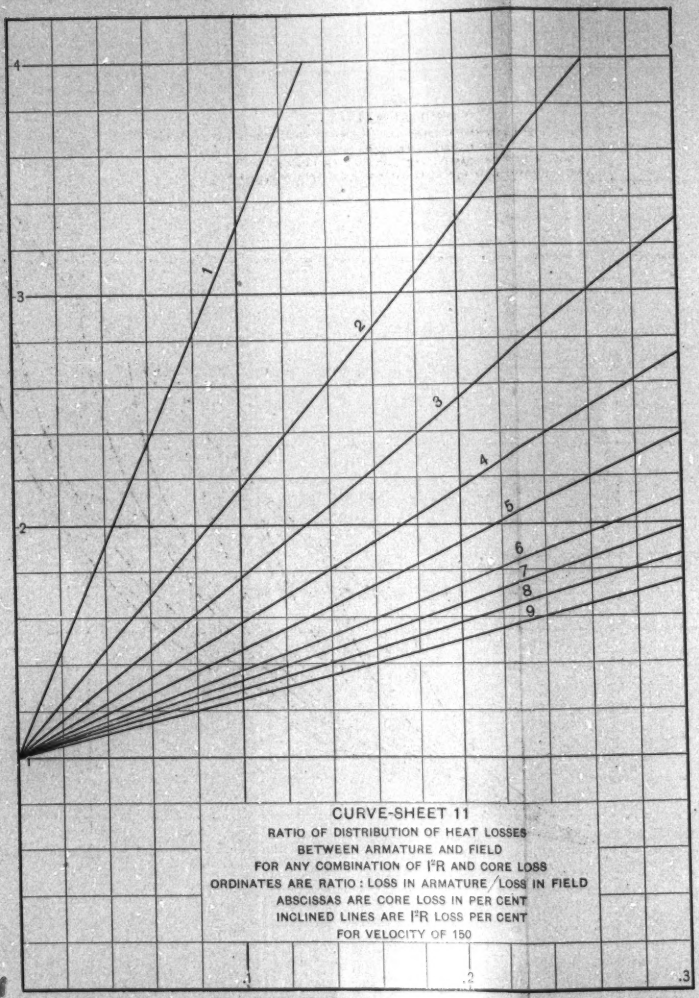

CURVE-SHEET 3
ENERGY INPUT IN WATT-HOURS PER TON-MILE
AND THROUGH ACCELERATION (A)
FOR DIFFERENT VALUES OF INITIAL ACCELERATION (α)
ACCELERATING TO A VELOCITY OF 150 ON THE MOTOR-CURVE
WATT-HOURS
250
200
150
100
50
.2
.3
.4
A
NO COASTING
α = 0.8
α = 1.0
α = 1.25
α = 1.5
α = 1.75
α = 2.0
α = 2.5
α = 3.0


CURVE-SHEET 4
MOTOR-CAPACITY
IN HORSE-POWER OUTPUT PER TON WEIGHT
AND THROUGH ACCELERATION(A)
FOR DIFFERENT VALUES OF INITIAL ACCELERATION(α)
ACCELERATING TO A VELOCITY OF 150 ON THE MOTOR-CURVE
H.P./TONS = Y × (LENGTH OF RUN IN MILES)^1/2
Y
A
40
30
20
10
.2
.3
.4
.5
α=3.0
α=2.5
α=2.0
α=1.75
α=1.50
α=1.0
α=0.8
α=0.6
NO COASTING




**CURVE-SHEET 10**

AVERAGE HEAT-LOSS DURING ACCELERATION
TO VELOCITY OF 150 ON MOTOR-CURVE
ORDINATES ARE AVERAGE LOSS IN PER CENT
ABSCISSAS ARE CORE LOSS " " "
INCLINED LINES ARE $I^2R$ LOSS " " "

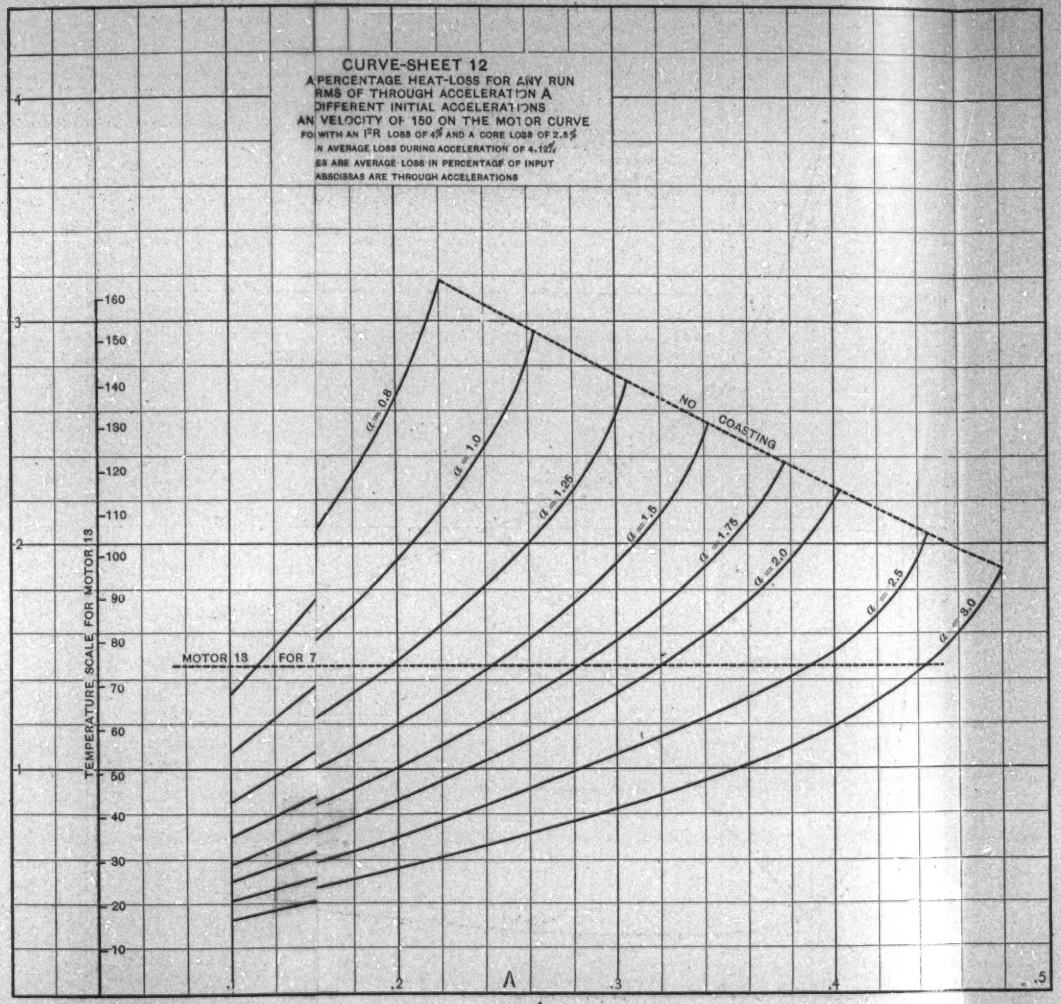



CURVE-SHEET 11

RATIO OF DISTRIBUTION OF HEAT LOSSES
BETWEEN ARMATURE AND FIELD
FOR ANY COMBINATION OF $I^2R$ AND CORE LOSS
ORDINATES ARE RATIO : LOSS IN ARMATURE / LOSS IN FIELD
ABSCISSAS ARE CORE LOSS IN PER CENT
INCLINED LINES ARE $I^2R$ LOSS PER CENT
FOR VELOCITY OF 150

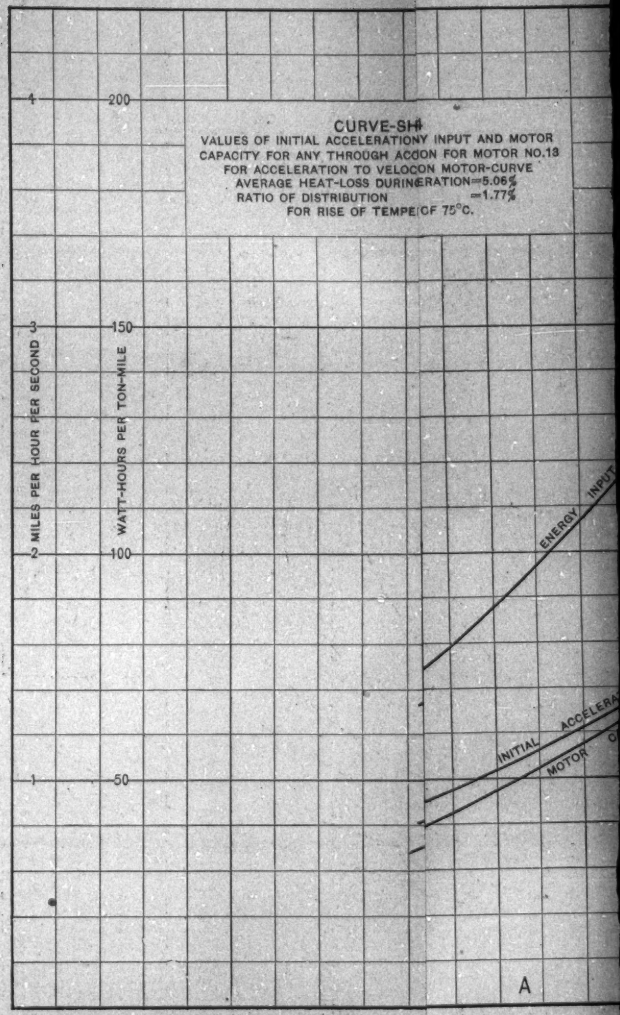
## CURVE-SHEET 12

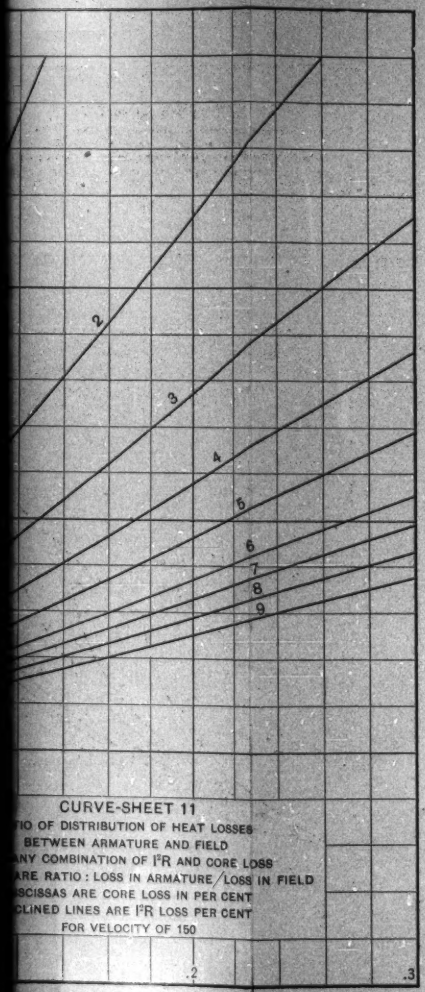
A PERCENTAGE HEAT-LOSS FOR ANY RUN
RMS OF THROUGH ACCELERATION A
DIFFERENT INITIAL ACCELERATIONS
AN VELOCITY OF 150 ON THE MOTOR CURVE
FO WITH AN $I^2R$ LOSS OF 4% AND A CORE LOSS OF 2.5%
N AVERAGE LOSS DURING ACCELERATION OF 4.12%
SS ARE AVERAGE LOSS IN PERCENTAGE OF INPUT
ABSCISSAS ARE THROUGH ACCELERATIONS




## CURVE-SH

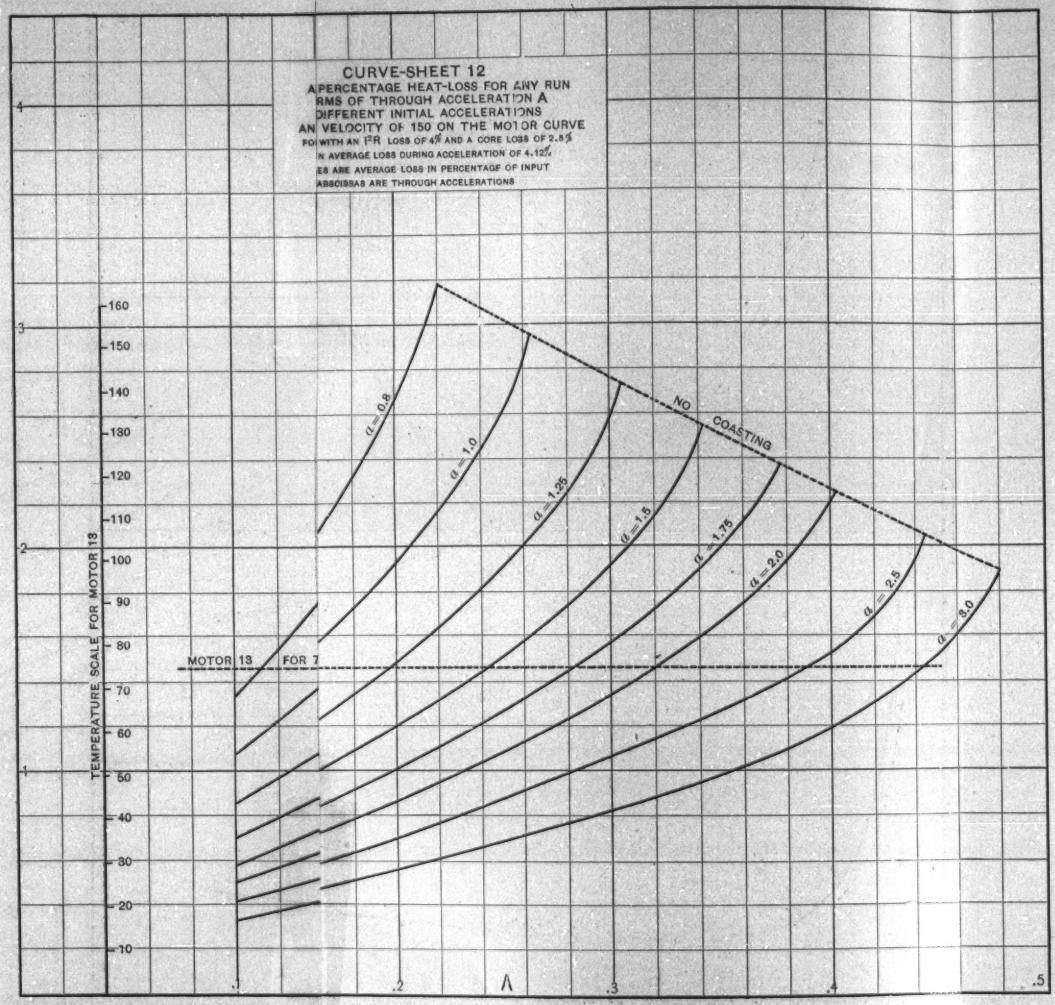
VALUES OF INITIAL ACCELERATIONY INPUT AND MOTOR
CAPACITY FOR ANY THROUGH ACCON FOR MOTOR NO.13
FOR ACCELERATION TO VELOCON MOTOR-CURVE
AVERAGE HEAT-LOSS DURINERATION=5.06%
RATIO OF DISTRIBUTION =1.77%
FOR RISE OF TEMPE OF 75°C.

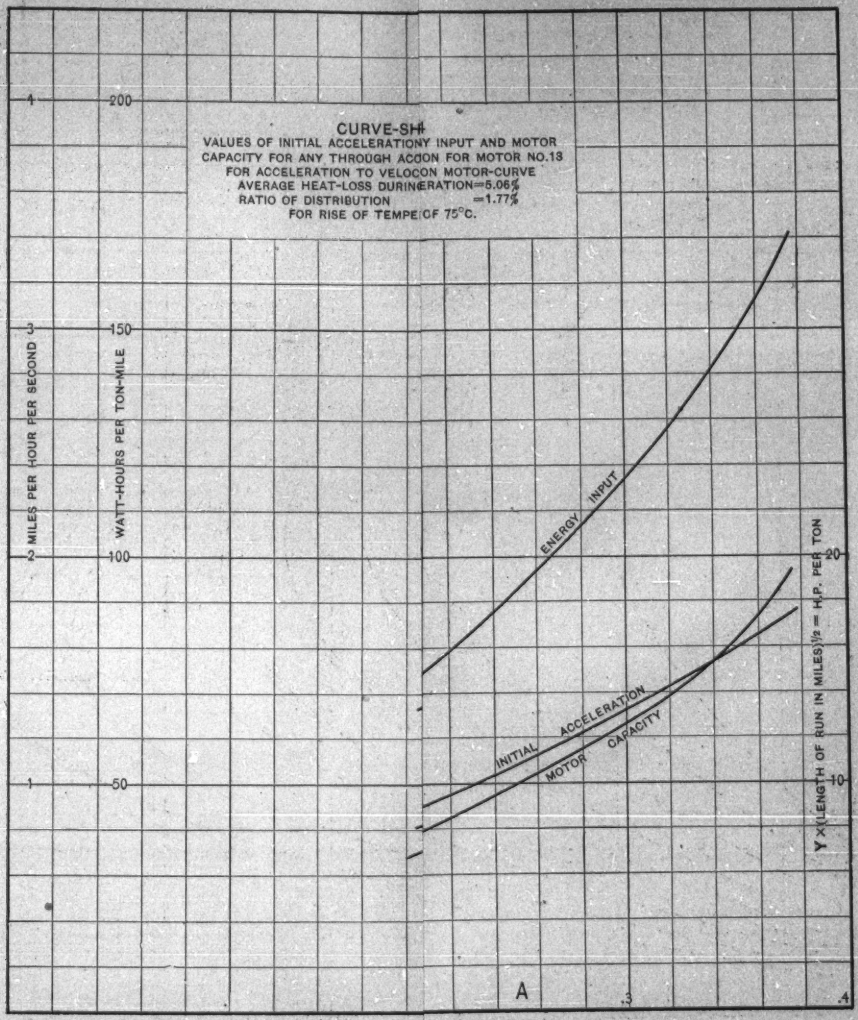





CURVE-SHEET 11

TIO OF DISTRIBUTION OF HEAT LOSSES
BETWEEN ARMATURE AND FIELD
ANY COMBINATION OF $I^2R$ AND CORE LOSS
ARE RATIO: LOSS IN ARMATURE / LOSS IN FIELD
BSCISSAS ARE CORE LOSS IN PER CENT
CLINED LINES ARE $I^2R$ LOSS PER CENT
FOR VELOCITY OF 150




CURVE-SH#
VALUES OF INITIAL ACCELERATIONY INPUT AND MOTOR
CAPACITY FOR ANY THROUGH ACCON FOR MOTOR NO.13
FOR ACCELERATION TO VELOCON MOTOR-CURVE
AVERAGE HEAT-LOSS DURINERATION=5.06%
RATIO OF DISTRIBUTION =1.77%
FOR RISE OF TEMPE(CF 75°C.
MILES PER HOUR PER SECOND
WATT-HOURS PER TON-MILE
Y X(LENGTH OF RUN IN MILES)½ = H.P. PER TON
ENERGY INPUT
INITIAL ACCELERATION
MOTOR CAPACITY
A
1
2
3
200
150
100
50
20
10
.3
.4

cs 4 gives the horse-power per ton for the same conditions; in the first paper, motor-capacity was given in kilowatt input; here it is reduced to horse-power output. These curves give the motor-capacity per ton required for any run expressed in terms of the maximum power input at the end of the initial acceleration.

An example will best explain their use: Suppose,

Schedule speed = 30 mph.
Distance = 7000 feet.
Time of stop = 25 sec.

Then

Time from start to start = 159 sec.
" " " " stop $-T$ = 134 "
Average velocity = $30 \times 159/134$ = 35.6 mph
Through acceleration, $A$ = $35.6/134$ = .266

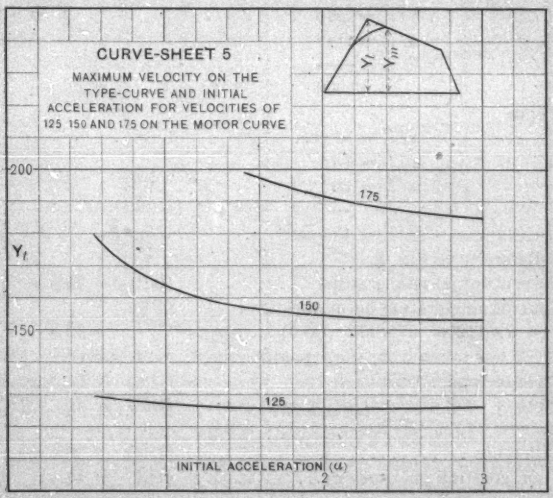
Suppose further that the speed, energy and motor-capacity are to be determined for

$$(a) = 1.25 \text{ mph/sec.},$$

and a velocity on the motor-curve of 150.

Then,

Relative maximum velocity on type-curve = 160 (CS 5).




(CS 5 is plotted from the velocity-time curves for the various initial accelerations).

$$x/T = .306 \qquad \text{(CS 1)}$$

Hence,

$$ax/T = .306 \times 1.25 = .382$$

and

$$ax = .382 \times 134 = 51.2 \text{ mph.}$$

This is the maximum velocity on the type-curve.

Hence,

Maximum velocity on motor-curve $= 150/160 \times 51.2 = 48$ mph

and

Velocity at resistance out $= 100/150 \times 48 = 32$ mph.

hence,

Velocity coefficient $= .32$

(Velocities are uniformly expressed in terms of the velocity at maximum power, as 100; the actual velocities are then equal to $\beta$ 100, where $\beta$ is a coefficient determined as indicated here, or in the manner explained in the first paper.)

Time of initial acceleration $= 32/1.25 = 25.6$ sec.

Usually it will not be necessary to determine the times required by the various parts of the run, but this can be done as follows:

$$x/T = .306 \qquad \text{(CS 1)}$$
$$y/T = .545 \qquad \text{(CS 1)}$$
$$z/T = .149 \qquad \text{(CS 1)}$$
$$1.000$$

Hence

$$x/T = 41 \text{ sec.}$$
$$y/T = 73 \text{ "}$$
$$z/T = 20 \text{ "}$$
$$T = 134 \text{ sec.}$$

Referring to Fig. 2:

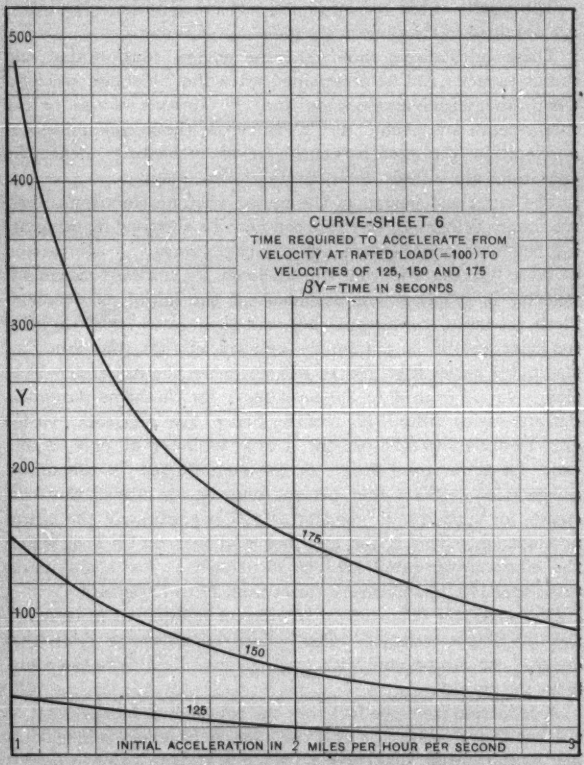
Time of initial acceleration.....(OP) = 25.6 sec.
Time of acceleration on motor-curve (PR)
$= \beta \times 112 = .32 \times 112$, (CS 6) .. = 35.8 "
(CS 6 is taken from the separate velocity-time curves.)
Time of total acceleration .....(OR) = 61.4 sec.
Time of total acceleration on type-run.....(OD) = 41 "

Difference .....(DR) = 20.4 sec.
Time coasting on type-run.....(DE) = 73. "

Difference, time coasting on motor-run....(RE) = 52.6 sec.

The various times are then,

| | | |
|---|---|---|
| Initial acceleration | (OP) | = 25.6 sec. |
| Motor acceleration | (PR) | = 35.8 " |
| Coasting | (RE) | = 52.6 " |
| Braking | (EC) | = 20. " |
| Total | (OC) | = 134.0 sec. |




It is noteworthy that this total, made up from readings taken directly or indirectly from half a dozen curve-sheets, should check exactly; of course such accuracy is partly accidental, but it shows how well graphical work of this character can be done.

The distances passed over for the various portions of the run can be calculated in a similar manner from other curve-sheets, which are not given here.

Further, from CS 4.

$$\text{Motor-Capacity} = 10.8 \left(\frac{7000}{5280}\right)^{\frac{1}{2}} = 12.5 \text{ hp/ton.}$$

Energy per ton-mile = 92 wh. (CS 3).

The results of the assumption of any other initial acceleration are obtained in the same manner.

These calculations show that the energy consumption and motor-capacity can be determined when the " through acceleration," the initial acceleration and the amount of use of the motor-curve are given. As I show below, the initial and motor accelerations are fixed by considerations of heating; hence the conditions are entirely determinate.

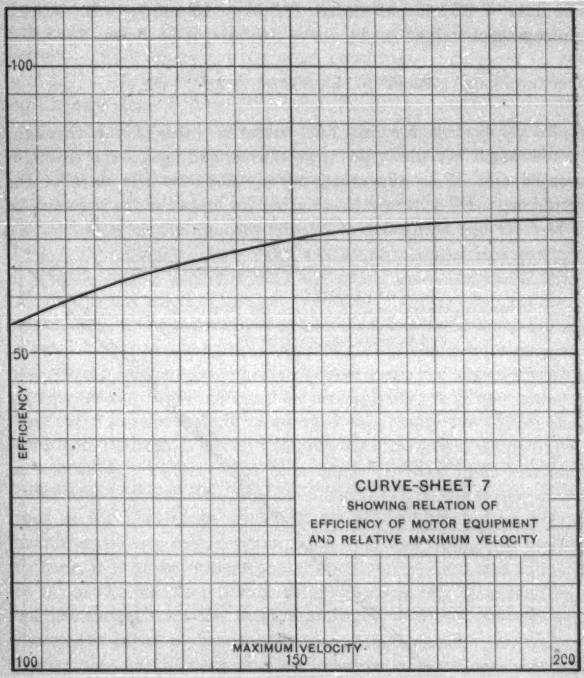
The input and output of the motor when accelerating to any maximum velocity at any rate can both be obtained by integrating the appropriate power and velocity curves. A comparison of these figures shows that the efficiency for constant maximum velocity is practically independent of the initial acceleration. Both the input and output vary, but their ratio varies slightly; for instance, for a maximum velocity of 125, the input of the motor varies from 804 to 653 wh. when the initial acceleration is varied from .6 to 3.0 mph/sec.; for the same range the output varies from 517 to 415; hence the efficiency varies only from 64.3 to 63.5 percent, a total variation of only .8 percent for a variation from .6 to 3.0 mph/sec. in the initial acceleration. Sixty-four percent may be considered the efficiency for a maximum velocity of 125, regardless of the initial acceleration. This is for the motor shown on CS 2, of which the efficiency at rated load is 85 percent. For a maximum velocityof 175, the efficiency varies from 71 to 73 percent.

CS 7 gives the efficiency of this motor equipment in terms of the maximum velocity. The values of efficiency from this curve-sheet can be used with small error to determine the input from the output.

A deduction from the fact that the efficiency is independent of the initial acceleration is that the average heat-loss is independent of the initial acceleration, which is also deduced directly. It is true that in the losses, friction loss is included, but the average friction-loss from zero to a fixed maximum velocity will not vary with the inital acceleration; hence the difference,

representing the heat-loss, must be constant to the same degree of approximation that the efficiency is constant.

The formula following gives the work done at the car-axle per ton-mile for various schedules, in terms of the through-acceleration and the proportional part of the total time used in braking. This formula and the efficiency from CS 7 permit the easy determination of the energy-input per ton-mile.




The formula for work per ton at the car-axle is deduced by calculating the work of friction for the entire distance, and the kinetic energy at braking; by the substitution of

$$L = .682\,AT^2$$

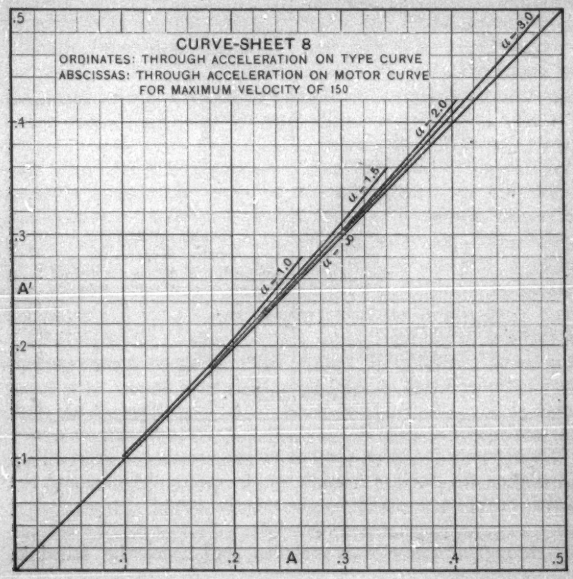
the through-acceleration is included, and hence the schedule. The formula is:

$$\text{Work per ton-mile at car-axle} = 180\,b + 90\left(c - b\right)\frac{c}{A}\left(\frac{z}{T}\right)^2 \text{ wh.}$$

The first term, $180\,b$, gives the work of friction per ton-mile; the other term, the energy wasted in brakes per ton-mile. $z/T$ is a function of the three accelerations, ($a$), ($b$) and ($c$), as well as of the through-acceleration. Hence the values of $z/T$ to be used in this formula must be obtained from curve-sheets plotted for the particular values of ($b$) and ($c$) that are substituted in the formula. CS 1 is calculated for ($b$) = .15 and ($c$) = 2; substituting these values in the above formula, it becomes,

$$\text{Work} = 27 + 333/A \times \left(\frac{z}{T}\right)^2 \text{ wh.}$$

To apply this formula, find first the value of the through-acceleration on the type-curve corresponding to the through-acceleration $A$ on the motor-curve, given by the data of the problem. CS 8 gives this relation for a maximum velocity of 150. Having this value of $A^1$, the through-acceleration on the



type-curve, select from CS 1 the value of $z/T$ corresponding to that value of $A^1$ and of the initial acceleration in question. For example,

For $A = .266$
and velocity $= 150$

CS 8 shows that $A^1$ equals .275. The value of $z/T$ for this value of $A^1$ and for an acceleration of 1.5 mph/sec., from CS 1 is .15, substituting in the above formula the work per ton-mile at the car-axle is found to be equal to 56 wh.; the efficiency, from CS 7 is 70 percent; hence the energy-input is 80 wh. CS 3 shows for the same conditions; *i.e.*, for $A = .266$ and $(a) = 1.5$, an energy-input equal to 83 wh., an agreement sufficiently close.

In the first paper, I assumed that an electric tramway motor in service will carry, on an average, a heat loss of 3 percent of its "hour-load," with a temperature elevation of 75° C. This is a fair average figure, but individual motors differ greatly in their heating characteristics; hence I have elaborated the method so that it may be used for motors having any copper-loss and any core-loss, for any schedule with any initial acceleration.

For this purpose I determine the ratio of average loss to loss at rated load for the copper-loss and core-loss separately, and for the several initial accelerations; knowing then the ratio of average loss during acceleration to loss at rated load, the average loss for any particular motor is found by multiplying the proper value of this ratio by the loss at rated load, separately for copper- and core-loss; the results added will give the total average loss during acceleration. These ratios of average to maximum loss were determined for velocities of 125, 150 and 175 on the motor-curve; that is, for maximum velocities 25, 50 and 75 percent greater than the velocity with resistance out. In describing this method further, it is to be understood that this procedure was followed in all cases; in other words, three separate sets of curves were carried through for the three maximum velocities, for the heat losses, for ratio of distribution, for motor-capacity, for energy-curves, and for the various other intermediate curves which were used as steps to the final results.

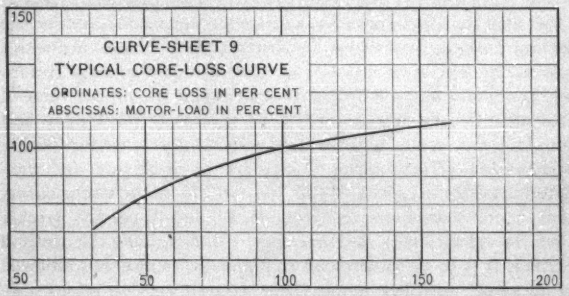
The rise of temperature of the parts of a motor, as determined by tests, depends upon the ratio of the distribution of the total heat-loss between the armature and field; to apply such test results correctly this ratio must then be known for the various conditions; hence the ratio of distribution was calculated for

the several initial accelerations and maximum velocities; the average loss during acceleration and its distribution having been determined in this manner, the average loss during the run will be known when the ratio of time of acceleration to time of run, for the various initial accelerations, maximum velocities and for all values of the through acceleration is known.

Having then the average loss during the run and its distribution between armature and field, the rise of temperature of a motor, used under the conditions of service fixed by any particular ratio of distribution, can be determined by comparing this calculated average loss with the rise of temperature per watt lost, for the same ratio of distribution, as determined by tests in service.

These steps required the preparation of nearly a hundred curve-sheets and some four hundred curves; the matter is too voluminous to present here in detail; I shall therefore give only the results and call attention to a few of the principal steps.

The ratio of average to maximum copper-loss is definite and was determined in the usual manner by plotting values for the current and integrating.




The ratio of average to maximum core-loss depends on the variation of core-loss with speed and current; to fix this variation, the individual core-loss curves of a number of motors were plotted, and a curve determined showing the average variation of the core-loss. CS 9 shows the curve of the average variation of core-loss used in this investigation; the values from this curve were plotted on the various velocity-time sheets and integrated; this gives the core-loss during acceleration on the motor curve. In addition to this, core-loss curves of several

motors, differing widely, were selected, and the ratios of average to maximum for different operating conditions were determined. These ratios were found to differ comparatively little for the different curves, much less than the actual values of the core-losses differ. The average core-loss during the period of acceleration on resistance was taken at 42 percent of the maximum core-loss. This value was obtained from a number of separate core-loss curves.

The average percentage heat-loss during acceleration to the same maximum velocity for any motor is practically independent of the initial acceleration; the ratio of average to maximum copper-loss increases with the acceleration, while the ratio for the core-loss decreases.

The variation of the loss with initial acceleration in two typical cases is given in the tables following:

TABLE I.

Average heat-loss, for velocity of 150.
Copper-loss at rated load = 3%
Core " " " " = 3%

| Initial Acceleration | Average Loss. |
|---|---|
| 1 mph/sec. | 3.88% |
| 1.5 " | 3.91 |
| 2. " | 3.91 |
| 2.5 " | 3.91 |

TABLE II.

Average heat loss, for velocity of 150.
Copper loss at rated load, = 7%
Core " " " " = 1%

| Initial Acceleration. | Average Loss. |
|---|---|
| 1.0 mph/sec. | 4.77% |
| 1.5 " | 4.92 |
| 2.0 " | 5.00 |
| 2.5 " | 5.03 |

For velocities greater than 150, the variation is greater; for lower velocities it is less. The values for $(a) = 1.5$ are used.

Although this approximation is satisfactory for the purpose, yet if greater accuracy is desired, it is possible to plot the average loss for the various initial accelerations separately; but the complication is very much increased.

CS 10 gives the average percentage heat loss during acceleration for any combination of copper- and core-losses within the range covered, for a velocity of 150 on the motor-curve. The ordinates are average loss, the abscissas core-loss, and the inclined lines copper-loss; for instance, for a 4.0 percent copper-loss and 2.5 percent core-loss, the intersection of the vertical at 2.5 with the inclined line marked 4, gives the average loss of 4.12 percent, and this will be the average loss occurring in a motor accelerating by series parallel, and then on the motor-curve to a velocity 50 percent greater than the velocity when resistance is cut out, the motor having a 4 percent copper-and 2.5 percent core-loss; the capacity being determined by the product of the torque during initial acceleration and the velocity with resistance out, that is the maximum power input.

The ratio of distribution is practically independent of the initial acceleration for a given maximum velocity, and for the range of initial accelerations used, *i.e.*, from $(a) = 1$ to $(a) = 3$. While this is not strictly correct, yet the difference between the values calculated accurately and the values found by assuming an average ratio of mean to maximum copper and mean to maximum core-loss, does not exceed 5 percent for the range covered; since this ratio is used only to fix the appropriate value of the radiation-constant (of which the determination is in itself necessarily rough) the degree of accuracy is sufficient.

CS 11 gives, for a velocity of 150, the ratio of distribution as ordinates, with-core loss as abscissas, and with inclined lines for the various copper losses; for instance, with copper-loss of 4.0 percent and core-loss of 2.5 percent, the ratio of distribution is 2.44. It is assumed that the copper-loss is half in armature and half in field.

These curves then determine the average loss during acceleration and its distribution; to get the average loss during the run the ratio of time of acceleration to time of run must be known.

Referring to Fig. 3,

$$R\,H/O\,R = A\,D/O\,D$$

*i.e.*,

$$(100+a\,t_2)/Y_t = (t_1+t_2)/x$$

and

$$(100+a\,t_2)/Y_t \times x/T = (t_1+t_2)/T$$

= Time of acceleration divided by total time.

$x/T$ is taken from CS 1 for the various values of $A$ and $(a)$, thus the ratio of the times is determined for all conditions; the corrections to the type-curve must be allowed for.

CS 12 has been prepared in this way; it gives the average percentage loss during the run, for various initial and through-accelerations, for a motor having a copper-loss of 4.0 percent, a core-loss of 2.5 percent, and consequently an average loss during acceleration to a velocity of 150, of 4.12 percent The ordinates are average losses in percentage of maximum input; the abscissas are through-accelerations and the separate curves are for the different initial accelerations.

The average loss during any particular run is equal to the average loss during acceleration multiplied by the ratio of the times, and since this ratio is independent of the losses in the

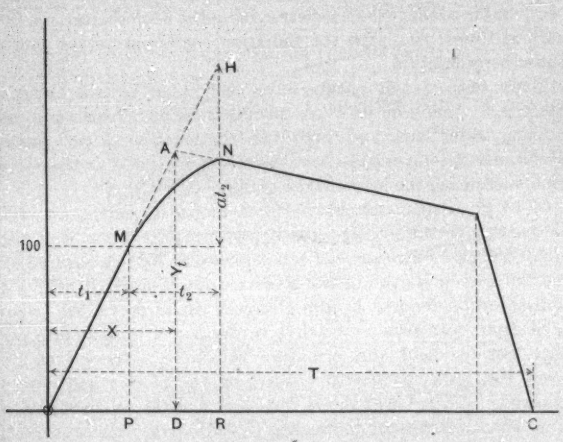



FIG. 3

motor, it is obvious that CS 12 will represent the average loss during that run for any motor, if the ordinates are varied in proportion to the average losses during acceleration. CS 10 gives the average losses during acceleration; hence these two sheets will determine the average losses for any motor for any run, accelerating to a velocity of 150 on the motor-curve. For example, a motor with a 9 percent copper and 2 percent core-loss has an average loss during acceleration of 6.7 percent; curve-sheet 12 is plotted for an average loss of 4.12 percent, hence if its ordinates be multiplied by $6.7/4.12 = 1.63$, it will give average loss for the second motor.

For example,

For $(a)$ = 1.25 mph/sec.
and $A$ = .2 "
Average loss = 1.48% (CS 12)

and therefore

$$1.63 \times 1.48\% = 2.41\%$$

is the average loss or the second motor for the same run. Similarly for other combinations of copper- and core-loss.

These curve-sheets determine the average heat-loss of any motor for any run, and its ratio of distribution; it remains then to connect these calculated values with results of test made under comparable conditions of service, giving the temperature rise for various losses, *i.e.*, with the radiation coefficient of the motor under consideration.

Many tests of this nature have been made by the General Electric Company on its experimental track at Schenectady, by running trains back and forth over the track until permanent conditions of temperature have been attained, and at the same time, measuring the losses in the motors.

CS 13 gives such test results for a motor in general use; the ordinates give the average loss in percentage of input that the motor will carry with a rise of temperature of 75° C. measured by resistance under the conditions of service determined by the ratios of distribution denoted by the abscissas; these curves are calculated from test results which give the rise of temperature per watt lost for field and armature separately; the curves are given for tests made with covers on the motors and covers removed. Thus CS 13 shows that for a ratio of distribution of 2, the armature of motor No. 13 will carry, with covers off, a load giving a loss of 4.82 percent, whereas the fields will stand only 3.1 percent; but if this motor be used with covers on, then at the same ratio the armature will stand a load giving a 2.05 percent, and the fields a 1.88 percent loss, very nearly the same, as should be.

These test results are used as follows:

Motor No. 13 has

Copper-loss = 6.05%
Core- " = 2.10%

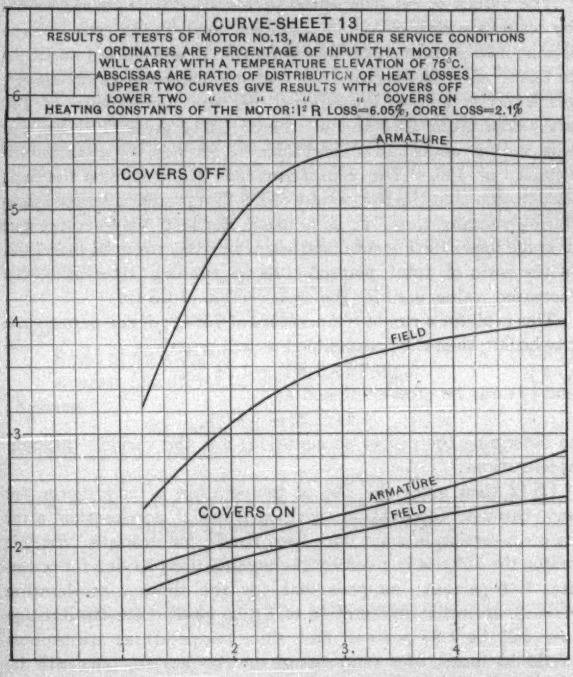
Then

Average loss during acceleration to velocity 150 = 5.06% (CS 10)
Ratio of distribution of losses = 1.77 (CS 11)

At this ratio of distribution, the permissible loss, with covers on, is

Armature, 1.98% (CS 13)
Field, 1.80% (CS 13)

Hence the field controls, and the motor can carry only a load giving a loss of 1.8 percent; the armature will then run much cooler than 75°; in fact, at $1.8/1.98 \times 75° = 68°$ C.




CS 12 is plotted for an average loss of 4.12 percent, hence to apply it to motor No. 13, the ordinates should be increased in the ratio of $5.06/4.12 = 1.23$: if this were done and a horizontal section of CS 12 were taken with the 1.8 per cent. line for ordinate, the initial accelerations would be determined, for each through-acceleration, that would result in a loss of just 1.8 percent and consequently in the exact temperature rise permitted;

but the same numerical results will be obtained by taking a cross-section of CS 9 at the ordinate $1.8\%/1.23 = 1.46\ \%$.

The dotted line marked "Motor No. 13" shows this cross-section. If the results of measurement by resistance, with covers off, be taken, the permissible loss for the field is 2.9 percent, by CS 13; for this condition the cross-section line should be drawn on CS 12 at $2.9/1.33 = 1.35$ percent.

The values of ($a$), the initial acceleration for measurements by resistance with covers off, as determined by the cross-section line at 2.35 percent (not shown on CS 12, are plotted on CS 14 in terms of $A$ as abscissa; the corresponding values of energy per ton-mile and of horse-power per ton for velocity of 150 are taken from CS 3 and 4. Thus, CS 14 gives the data for this motor under any conditions of service, for a maximum velocity of 150. The comparison has been made on the tacit assumption that the test results of CS 13 represent average losses during running time; if, as is usual, the test values represent average losses during total elapsed time, they must be increased in the ratio of total elapsed time to running time and this increased value used for the cross-section of CS 12.

This is all for a rise of temperature of 75° C.; if the permissible rise be 60°, then the permissible loss will be only

$$60/75 \times 1.8\% = 1.44\ \%,$$

and 12 must be cross-sectioned at

$$1.44/1.23 = 1.17\%,$$

to determine the proper accelerations for the various through-accelerations.

CS 12 then gives the rise of temperature of any motor for any through acceleration, with any initial acceleration, after its cross-section line has once been fixed; for example, for this motor the 75° rise is given by the accelerations having for ordinates 1.46 percent; suppose that for the through-acceleration $A = .2$, an initial acceleration of 1.0 be used; then the average loss will be 1.93 percent, and since temperatures are proportional to losses, the temperature of the hottest part will be $1.93/1.46 \times 75° = 99°$ C. If an initial acceleration of 1.75 mph/sec. be used, the rise of temperature will be $1.00/1.46 \times 75° = 51°$ C.

That is, for any motor and any value of $A$, the ordinates of CS 12 will give the rise of temperature for any initial acceleration; each initial acceleration corresponds to a definite rise of temperature for each through-acceleration; a vertical cross-section of CS 12 at any value of $A$ shows the relation of rise of temperature and initial acceleration for that through-

acceleration. The dotted vertical line on CS 12 gives the temperature scale for motor No 13, for $A = .25$ and $(a) = 1$, the rise will be 134°; for $A = .15$ and $(a) = 2$, the rise will be 40.5°. The temperature scale can thus be drawn on CS 12 for any motor, knowing its heating constants.

In order to avoid confusion, it is essential to recall that each change of initial acceleration means a proportional change in motor-capacity, the motor-capacity being directly proportional to the product of initial torque and the rated velocity. It It is thus tacitly assumed that the engineer has a complete range of motors of all capacities, all with the same heating constants; but the results are given in horse-power per ton and hence they give merely a determination of the load that any given motor will carry, under the various conditions.

Returning to the example given on page 6, where the initial acceleration was assumed to be 1.25 mph/sec.; if the heating constants of the motor to be used are known, the initial acceleration will be fixed; assume the motor for this run to have these constants:

$$I^2R\text{-loss} = 3.6\ \%$$
$$\text{Core-loss} = 2.75\%$$

Then

Ratio of distribution = 2.8 (CS 11)

Average loss during acceleration = 4.06% (CS 10)

Assume,

Radiation at ratio of 2.8 = 1.6%

Then a horizontal section of CS 12 having as ordinate

$4.12/4.06 \times 1.6\% = 1.62\%$ gives,

for $A = .266$ – the through acceleration under consideration, –

Initial acceleration = 1.5 mph/sec.

and

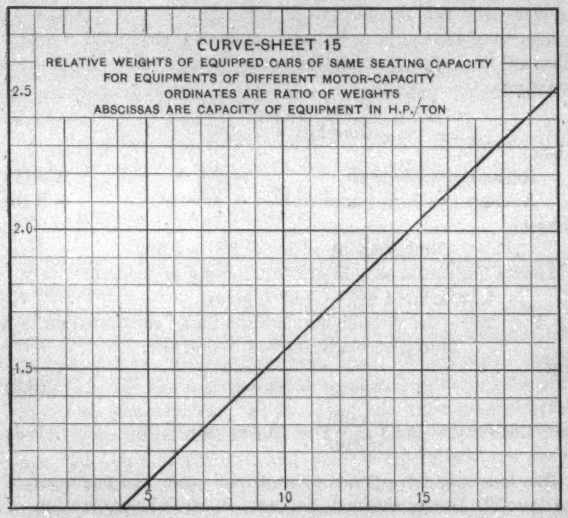
Energy = 83 wh. (CS 3)

$$\text{Capacity} = 11.6\left(\frac{7000}{5280}\right)^{\frac{1}{2}} = 13.3\ \text{hp/ton}. \qquad \text{(CS 4)}$$

This example corresponds to an actual case; the engineers of the project have decided to use an initial acceleration of 1.5 mph/sec., and expect to obtain an energy consumption of 87 wh. per ton-mile; these figures agree closely with those above.

CS 14 gives what may be called the "characteristic performance" of Motor No. 13, used for any schedule as determined by resistance measurements with covers off. It is obvious that curve-sheets similar to this can readily be plotted

for maximum velocities of 125 and 175. A comparison of such curve sheets for the different maximum velocities will, in general, show slight variations in motor-capacity for different initial accelerations, for a given schedule. In some cases the minimum motor-capacity is attained with an average maximum velocity, say, from 140 to 150; in other cases the motor-capacity diminishes as the maximum velocity is increased, up to the limit used in this investigation. No general conclusions can be drawn for the particular relative maximum velocity to be used; this depends principally upon the heating constants of the motor, including the radiation. It is clear, however, that by the aid of curves such as CS 12 for the various maximum velocities, the minimum motor-capacity can be found for any particular motor; all that is necessary is to draw the cross-section lines at the proper




points, thus fixing the value of the initial acceleration, and then to select the values of energy and horsepower from the different curve-sheets. In the case of Motor No. 13, CS 14 for velocity of 150 gives practically the minimum motor-capacity; there is a diminution for greater maximum velocities, but it is comparatively unimportant.

In all cases, the particular initial acceleration leading to the minimum motor-capacity should be adopted, even though this initial acceleration does not give minimum energy. The curves give the energy required per ton of total weight; hence the energy required per car is proportional to these figures multiplied by the relative weight of the car. The difference in the energy per ton for any schedule, is comparatively slight for the different maximum velocities, and in all cases the energy required per car is a minimum for the minimum motor-capacity with which the schedule can be made; the increase in the weight of equipment much more than balancing the diminished energy per ton obtained by using higher initial acceleration and higher motor-capacity. CS 15 gives the ratio of weight of equipment of different capacities; it is substantially the same as CS 15 of the former paper. The energy per ton multiplied by the ordinates of this curve-sheet gives relative figures showing the energy per car.

By this method, the two quantities that have, up to this point, been matters of arbitrary choice, are determined, *i.e.*, the initial acceleration and the relative maximum velocity on the motor-curve. Having then a motor with known heating constants, all the conditions which should govern its use for any particular schedule are fixed, and consequently the difference in the results attained by the use of motors with different heating constants is also determined.

I have illustrated the application of this method in comparing the availability of motors with different heating constants for a particular service. It is clear that it can also be used to obtain the data for different schedules. As an example of this use, suppose the data for various schedule speeds for a run of 7,000 feet, with twenty-five second stops, are to be determined

Assume the motor represented by CS 14; then Table III. will show the performance of this motor for the various schedule speeds. In this Table, columns 1, 2 and 3 are calculated from the length and time of stop; columns 4, 5 and 6 are taken from CS 14; column 7 gives the ratio of the numbers of column 6, taking 7.4 as 100; column 8 is taken from CS 15; column 9 gives the relative total energy required per car; it is obtained by dividing the product of the figures of columns 5 and 8 for each schedule speed, by the corresponding product for a schedule speed of 25 mph. As the weight ratios of CS 15 are for a motor-car of the same size, but with different motor equipments, the figures of column 9 give the relative energy

requirements per car-seat or per passenger for the different schedule speeds. Column 7 of this table may be taken as a sort of rough indication of the relative cost of the equipment for the different schedule speeds, and column 9 of the relative operating costs. They illustrate forcibly the great increase required both in motor equipment and in energy consumption for comparatively small increases in schedule speed.

TABLE III.

| Schedule Speed | Average Speed. | Through Acceleration. | Initial Acceleration. | Wh/Ton. | Hp./Ton. | Relative Motor-Capacity. | Relative Weights of Equipment. | Relative Total Energy. |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 25. | 28.8 | .174 | .77 | 68 | 7.4 | 100 | 1.49 | 100 |
| 27.5 | 32.1 | .216 | .98 | 77 | 9.4 | 127 | 1.65 | 137 |
| 30 | 35.6 | .286 | 1.15 | 99 | 12.3 | 166 | 1.80 | 199 |
| 32.5 | 39.1 | .330 | 1.42 | 127 | 15.9 | 215 | 2.37 | 320 |
| 35 | 42.9 | .386 | 1.84 | 180 | 21.0 | 324 | 2.92 | 560 |

The curve-sheets show initial accelerations as high as 3 mph/sec., and through-accelerations up to $A = .4$; these values are beyond the range of practice, both on account of the excessive motor-capacity demanded and the close approach to the slipping point of the wheels, with the usual percentage of total weight on drivers. The boundary lines of the curve-sheets can fairly be taken as

$$A = .1 \text{ to } A = .3$$

and

$$(a) = .8 \text{ to } (a) = 2.$$

This indicates the comparatively narrow limits within which all schedules are brought by the use of the quantity $A$.

It is obvious that the methods outlined here can be applied to alternating current motors by the use of appropriate speed-torque curves in place of those of CS 2.

The method which has been developed takes account of all the factors governing the rise of temperature of motors under conditions of service. While certain assumptions have of

necessity been made—as, for instance, a constant value for train-friction—it is perfectly feasible to re-calculate the curve-sheets with other values of train-friction; a set of curve-sheets similar to CS 12, including as many maximum velocities as may seem desirable, can be prepared and used as standard curves from which the rise of temperature of any motor can be determined very simply, for any conditions of service.

While the results obtained by this method are of necessity an approximation because of the number of assumptions that it is necessary to make in developing the method, yet numerous applications of the curve-sheets to practical examples have shown that the approximation is close, and that the curves furnish a useful means for determining the performance of any motor in service.

## Discussion.

PRESIDENT ARNOLD stated that he believed those present had been much interested in Doctor Hutchinson's paper as it supplemented the paper the author had previously delivered before the INSTITUTE: that some of the members, when solving railway problems, had found that some of the previous conclusions of the author were nearer correct than some who discussed the paper were willing to concede at the time it was read. That as the railway motor problem was becoming more important as the size of motors increased he was glad the author had seen fit to supplement his former paper with the present one and discuss more fully the relationship of heat upon the capacity of motors.

MR. A. H. ARMSTRONG:—The paper presented by Doctor Hutchinson is of such a broad scope that it requires careful study before intelligent discussion is possible, and my remarks are therefore limited to one or two aspects of the problem. The author has adopted a method, in common with others, of following through the performance of the motor in practice, segregating the losses, proportioning them between armature and field, getting their ratio and subsequently connecting these results with the temperature rise by means of the actual performance of the motor determined by a series of experimental tests. Various papers have been brought out on the subject, notably that of Mr. Mailloux, which very carefully considered a method of plotting speed-time curves, and while the paper of Doctor Hutchinson has not given much space to this detail it has attempted to bring out a broad method of arriving at the service capacity of railway motors. There are one or two points which the author has not touched upon, perhaps not considering them necessary in the rather broad manner in which the subject has been taken up. The first of these is the neglect to include in the speed time and ampere-time curves the effect of the energy lost in accelerating the rotating parts of the train. In slow-speed work, that is, where the maximum speeds do not much exceed 30 mph., the energy required to rotate the motor armature, gears, wheels, axle, etc., may amount to a possible 15 percent of the total accelerating energy required for the train. Problems similar to the operation of trains on the New York Elevated would come in this class. In suburban work where cars reach a maximum speed of 60 mph. or more, the energy demanded to accelerate the rotating parts will not be more than 4 or 5 percent of the energy input to the train during acceleration. As both of these percentages are given in terms of energy input during acceleration and do not consider total input for the completed run, it is evident that leaving them out would not introduce a very large error, but the inaccuracy would be greater at low speeds than at high speeds.

The paper is based upon the assumption of a constant friction-rate, the author assuming 13.6 lb. per ton for all speeds. As the

problem is worked out for only this one friction-rate, the conclusions arrived at in the final curves cannot therefore in any way be assumed as general in their application, due to the fact that train-friction varies greatly with the speed, especially in single car operation. In fact, a friction-rate of 13.6 lb. per ton would not have a field of application outside of the Rapid Transit problems involving the use of four or five cars in a train and operating at speeds not greatly exceeding 30 mph. maximum. If the problem is to be treated in a broad way, it is necessary to assume a number of friction curves, agreeing with the number of cars in a train, their shape and the load which they carry, and determine motor-capacities for all these different friction-curves. It would be sufficient to plot motor-capacities on the basis of three friction-curves and interpolate for friction-values lying within their scope. This has not been done and it leads to some inaccuracy in some of the conclusions arrived at in the paper. For example, the paper is based upon maximum speeds reaching 25, 50 and 75 percent above the speeds at which starting resistances are cut out. Based upon an assumption of 13.6 lb. per ton it is possible to get these different maximum speeds, but from tests which the General Electric Company has made and which have been corroborated by other authorities, it has been found that the friction of single cars of light construction sometimes reaches 50 or 60 lb. per ton when operating at speeds of 60 mph. or more. One of the examples given in the paper mentions a maximum speed of 50 mph. so that the author evidently had in mind the universal application of his curves, an assumption which is obviously incorrect considering that they are all plotted for a constant friction of 13.6 lb. per ton.

It is interesting to note the maximum speeds which it is possible to reach with a given motor when operating trains of different composition, giving different friction-rates. For example, a single car operated at a maximum speed of 30 mph. with a motor of modern construction having a rational speed-torque curve makes it possible to reach a maximum speed which is double that at which starting resistance is cut out; in other words, expressed in the terms of the author's paper, it is possible to obtain a value of 200 under these conditions. For two cars operating at the same speed, it is possible to obtain a maximum speed of 226, while for a train of heavy cars it is possible to reach a maximum speed of 250. It would therefore be possible to obtain the maximum speeds of 125, 150 and 175 mentioned in the paper, provided the maximum speed of the train expressed in miles per hour did not greatly exceed 30. For the higher friction-rates obtaining in higher speed service, that is possibly as high as 75 mph., the maximum speeds mentioned by the author are not possible. For example, a single car operating at 75 mph. could hardly exceed a maximum speed of 123, assuming the speed at which starting resistances are cut out to be 100. Two such cars make it possible to reach a maximum speed of 140,

and for a train of heavy cars giving lowest friction-rate obtainable a maximum speed of 190 could possibly be reached. If the conclusions are based upon a maximum speed of 150 and the paper carried through on this basis, the results would not apply to universal railway work, especially when high-speed problems are encountered. The results arrived at in the paper are applicable therefore more especially to rapid transit problems similar to the New York Elevated, but would not give accurate results for all classes of service.

On page 1570 the author gives a table showing the motor capacities required for certain runs under given conditions. Some results obtained from experimental tests of motors are at hand giving the energy-consumption, and it is very interesting to compare the results obtained by attacking the problem from different standpoints and by different investigators. The results obtained so far as they relate to the relative capacity of motor required check faily well as the following table indicates. The figures given below relate to the column 5 on the table given on page 1570, which details the watt-hours per ton-mile required for the conditions named. Column A is quoted from the table and Column B from experimental results.

| A. | B. |
|---|---|
| 63 wh. | 50 wh. |
| 77 | 60 |
| 99 | 73 |
| 127 | 102 |
| 180 | 125 |

Column 7 in the same table gives the relative motor capacity required for a temperature rise of 75° C. above the surrounding air. As in the previous table, column A is quoted from the table and column B from experimental results.

| A | B |
|---|---|
| 100 | 100 |
| 127 | 125 |
| 166 | 156 |
| 215 | 212 |
| 324 | 281 |

These figures also show a fair agreement. Comparing column 6 with experimental results, the same agreement does not hold as shown in the following table. This column is understood to give the horse-power rating of the motors based upon a stand test which will permit a rise of 75° C. above the air in the hottest part of the motor. Column A is quoted from the table and column B from experimental results.

| A | B |
|---|---|
| 7.4 | 5 5 |
| 9.4 | 6.9 |
| 12.3 | 9.3 |
| 15.9 | 11.5 |
| 24. | 14.5 |

The figures quoted in the table of the paper seem to be some 50 percent greater than the value in column B obtained from experimental results.

These figures all contemplate a fixed friction-rate of 13.6 lb. per ton. It has been found that with the friction-rate obtained at speeds of 60 mph. or more, the motor-capacity demanded will increase fully 20 percent more than the values indicated by the flat friction-rate of the author's paper. This percentage increase will depend upon the composition of the train, whether a single car is operated alone or several cars are operated in a train. In other words, the high-speed problem must be treated upon the basis of several friction-curves plotted to meet the different operating conditions, using that curve for obtaining the motor-capacity and energy-consumption which applies to the case in hand. The energy-consumption at high speeds is very greatly affected by the rate of friction assumed. In slow-speed work, to which the paper as given must be confined owing to the assumption of 13.6 lb. per ton friction-rate, the energy demanded in accelerating the train is very considerable compared with that required to overcome friction, and hence the latter is of secondary importance as affecting the energy-consumption. At higher speeds, however, approaching 60 mph. or more, especially when single cars are operated at these speeds the friction-rate determines the energy-consumption, and the energy lost in accelerating becomes in turn of secondary importance. For example, with a friction-rate of 20 lb. per ton, which might be assumed for a train of several cars at 60 mph., the energy-consumption must necessarily be 40 watt-hours per ton-mile, or double the friction-rate, without introducing any energy-loss due to acceleration or heating of the motors.

On the other hand, a single car weighing 30 to 35 tons, operating at this same speed would have a friction-rate very close to 50 lb. per ton, demanding 100 watt-hours per ton-mile without any energy lost in accelerating the car or in the motor. This indicates the extreme importance of carefully determining by exhaustive tests what friction-rate should form the basis of calculations for the case in hand, and a broad treatise on railway motor capacity will be of value only when applicable to any friction-rate at any speed.

MR. CARY T. HUTCHINSON:—Mr. Armstrong's remarks seem to require some explanation in order that this matter may be made clear. In general, Mr. Armstrong attempts to apply results from a particular example, illustrating a general method, to conditions under which they are clearly not applicable. It is distinctly stated that the paper is intended to develop a method of treating this problem; the numerical results given are merely illustrations; nevertheless, these numerical results should be in agreement with results obtained by other methods, *under identical conditions*.

Mr. Armstrong's first criticism is that a constant friction-rate

of 13.6 lb. per ton has been taken, and that such a rate is entirely too low for high maximum-speeds. This is, of course, correct; it is stated in the paper that some constant rate must be assumed and that if problems are to be solved involving a rate differing greatly from the rate assumed, new curve-sheets must be drawn, based on the proper friction-rate. The effect of a different friction-rate upon the energy required is easily determined by the equation on page 1558 of the paper, giving the work done. The constant, 27, in this equation is equal to the watt-hours per ton-mile for a friction-rate of 13.6 lb. per ton; the second term of that equation gives the additional energy required for the various schedules. If any other friction-rate than 13.6 be used, the work of friction will be increased approximately in that ratio.

It is not intended to deny the very great importance of using the proper friction-rate, particularly for high speeds; but only to point out that a criticism of the method based on this fact is not to the point, since in the paper itself the necessity for considering this fact is pointed out.

Mr. Armstrong seems to be of the impression that the results of the paper are based on accelerating to a maximum speed 50 percent greater than the speed at resistance-out. This of course is not the case. The paper states that curves were plotted for velocities of 125, 150 and 175, and that the minimum motor-capacity for any particular run is to be picked out from the results at these various velocities. It happens, however, that a maximum velocity of 150, in many of the cases examined, gave very nearly the minimum motor-capacity, hence curve-sheets for this particular value of the maximum velocity were presented, as illustrating average conditions better than any other value. The method, however, is applicable to any maximum velocity that it is possible to obtain with the train-friction assumed.

The next point referred to by Mr. Armstrong—*i.e.*, the possible maximum-velocity—is merely a question of the ratio of the train-friction to initial horizontal-effort, and therefore involves the initial acceleration used. With a train-friction of 13.6 lb. per ton, an initial acceleration of 1 mph/sec., and a motor so chosen that its rated torque will be approximately 105 lb., then after an infinite time the motor will come to a constant velocity, and the torque will be $13.6 \div 105 = 13$ percent of the rated torque. A reference to the speed-torque curve of the motor, such as Curve-Sheet 2, will then immediately determine the maximum velocity. In this particular case, the maximum velocity would be about 200 percent with the velocity at resistance-out as 100. A similar calculation will give the maximum velocity for any friction-rate. The higher the friction-rate and the lower the initial torque, the lower will be the relative maximum-velocity.

Mr. Armstrong's chief criticism is, however, that Table III. of the paper does not give results in close agreement with results obtained by his Company under practical conditions. Mr. Armstrong's figures give the horse-power rating of the motor

based upon a rise of 75° C. above the air, in the hottest part of the motor, as measured by *thermometer*. The figures in the table, however, are for a rise of 75° C. determined by *resistance-measurements*, with covers off, and for a track test. To illustrate the effect of conditions under which the tests are made, the following table is given; it shows the percent of input that this motor will radiate for a temperature-rise of 75° C., under the service-conditions denoted by a ratio of distribution of 1.8, when measurements of temperature are made by resistance, by thermometer, and with covers-on the motors and covers-off. The energy and power per ton for a run of one mile, taken from C.S. 12 for a through acceleration $A = 0.266$ is added.

TEST-RESULTS OF MOTOR 13 FOR RATIO OF 1.8.

| No. | Temperature Measured by. | Covers of Motor. | Radiation for 75° C. in percent of input. |
|---|---|---|---|
| 1 | Thermometer | On | 2.21% |
| 2 | Thermometer | Off | 4.55% |
| 3 | Resistance | On | 1.74% |
| 4 | Resistance | Off | 2.78% |

For $A = 0.266$ and a one-mile run

| No. | Initial Acceleration ($a$.) | wh/ton. | hp/ton. |
|---|---|---|---|
| 1 | 1.4 mph/sec. | 85 | 11.4 |
| 2 | Less than 0.8 | | |
| 3 | 1.72 | 78 | 13.2 |
| 4 | 1.20 | 94 | 10.8 |

That is, with resistance-measurement the necessary motor-capacity is 22 percent greater with covers-on than with covers-off. This is the measure of the effect of ventilation. The curves are not continued far enough to give results for thermometer and covers-off.

The principal object of the paper was to show how to handle this problem for motors having any copper- and core-loss; Mr. Armstrong gives average results without any reference to the particular motor for which they were obtained, and consequently without any reference to the heating-constants of the motor. This is an added reason why no direct comparison of numerical results is possible.

A study of the paper will show that the results derived from the curve-sheets obtained in this general way must be accurate, for the simple reason that they are based on the performance sheet of the motor itself, and introduce no possible errors other than those made in the assumptions given; the results will then be as accurate as are the experimental tests of the motor.

LOUIS DUNCAN (*by letter*):—The object of the paper seems to be the development of a systematic method for showing the effects of the different heat constants, *i.e.*, $I^2R$-loss, core-loss, and the radiation-constants upon the performance of railroad motors under service conditions. The $I^2R$-loss and core-loss may be obtained by shop tests, while the radiation-constants are determined by experimental track-tests under conditions imitating the problems to be solved. The author makes a number of assumptions in order to bring the problem within the limits of general treatment. For instance, he assumes a straight, level track with constant train-friction and constant braking-force. As far as the constant train-friction goes this assumption would not hold within the limits of train velocities used in the paper, but it is near enough to give approximate results, and more accurate results could be obtained by plotting the different curves on the basis of a train-friction varying with the speed of the train. The author also assumes that the type of motor-curve is the same for the different motors whose performance he considers, and states that the shape of the curves for the different motors is practically the same, but that the efficiency varies through a considerably wider range.

In a number of the curves in the paper the different quantities are potted in terms of a quantity "A," which the author calls "through acceleration." It should be constantly borne in mind in reading the paper that this quantity is deduced from the schedule speed and the length of run, these two quantities being introduced into the discussion by means of the quantity "A." Frankly, the writer does not consider that curves which show relations between real quantities, like acceleration, and fictitious quantities, like the quantity "A," are particularly instructive. Still, as a short method of arriving at the desired results, it has its uses. After plotting a number of curves giving the energy input in watt-hours per ton-mile and the motor-capacity per ton for different schedules, *i.e.*, for different values of "A," the author proceeds to introduce the heating-constants of the motors as determining their performance.

Curve-Sheet 10 in the paper connects the total percentage loss in the motor during acceleration with the percentage of $I^2R$-loss, and the percentage core-loss, these all being in percentages of the maximum input of the motor. It is assumed in plotting these curves that the maximum velocity is 50 percent greater than the velocity when all the resistance is cut out. Using another set of curves which give the ratio of the time of acceleration to the total time for different schedules, Curve-Sheet 12 is prepared which gives the relation between the percentage total loss for the entire run, the schedule, and the initial acceleration—all of these quantities being for a motor which has an $I^2R$-loss of 4 percent, a core-loss of 2.5 percent, and an average loss during acceleration of 4.12 percent, the maximum velocity being 50 percent higher than the velocity when all the resistance is cut out of the motor circuit.

This Curve-Sheet No. 12 may be used to determine the average loss during the given run for *any* motor, for while it is plotted for a motor having an average loss of 4.12 percent during acceleration yet if another motor had a loss of, say 5 percent, it would only be necessary to multiply the ordinates in the ratio of 5/4.12. If the temperature attained by a particular motor under service conditions for any particular loss is determined by track-tests, it is clear that the initial acceleration leading to exactly this loss can be picked out for any schedule from Curve-Sheet 12. The author gives illustrations of the method of using this curve-sheet and establishing a temperature scale for motors having different losses than those of the motor for which the curve is plotted. In other words, Curve-Sheet 12 determines for any schedule, what initial acceleration must be used in order that a motor may have any desired temperature elevation, when the copper- and core-losses of the motor are known and the value of the radiation-constants of the motor are given by tests under service conditions.

From Curve-Sheet 12 the author obtains performance sheets such as Curve-Sheet 14, which gives the initial acceleration, the motor-capacity, and the energy-input for the particular motor considered in Curve-Sheet 12 for all schedules, the temperature rise being 75°. Similar curves could be prepared for different temperature rises. It should be noted that these curves are based on a maximum velocity 50 percent greater than the velocity with all the resistance cut out of the motor circuit, but the curves given in the paper would allow Curve-Sheet 14 to be varied to correspond with different maximum velocities. If curve-sheets are made for different maximum velocities and compared with one another, that maximum velocity for a given schedule may be selected which will give the minimum motor-capacity; *i.e.*, the maximum tons per horse-power capacity of the motor for which the required schedule can be made. The author states, and the writer considers this important, that the best results are always obtained with the minimum motor-capacity, for while the energy per ton-mile may not be a minimum, yet the difference is not great for any schedule for different maximum velocities, while the weight of equipment increases rapidly as the initial acceleration increases, so that the energy *per car* is less when the equipment has the minimum capacity necessary to make the schedule. This fact was brought out in the author's first paper and is further emphasized in this one.

What the author has done, then, is to develop a method for obtaining for a given schedule and a given rise of temperature, the capacity in h.p. per ton, the energy-input, and the initial acceleration for any motor, taking as data the $I^2R$- and core-losses of the motor and the radiation-coefficients determined by actual tests of the motor under service conditions. While he has made a number of assumptions, some of which are not strictly true, yet the applications of the method show sufficient agreement with the actual facts to prove that the method will be useful for practical purposes.

*A paper presented at the 180th Meeting of the American Institute of Electrical Engineers, New York, Oct. 23d, 1903.*
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# A GRAPHIC RECORDING-AMMETER.

BY A. H. ARMSTRONG.

With the advance of the study of quick acceleration, the need was felt for an instrument capable of registering accurately current and voltage readings which fluctuate violently. Such an instrument must be capable of operating satisfactorily on a car moving at a high speed over bad track, and hence subject to vibration and shock. Sudden changes in the quantities measured required that the instrument should have a short period and should be perfectly damped. The swaying of the car, the rapid acceleration and retardation made it necessary that the instrument should be thoroughly balanced. In order to obtain a permanent record with freedom from sticking it was necessary to use ink, since even with the high torque of this instrument a metal stylus or graphite pencil-point would not give good service.

To secure the required torque, a dynamometer construction is used, in which the current to be measured is carried by the fixed coil and a constant current of small value flows through the moving coil, supplied preferably by a storage-battery.

The moving coil, rectangular in shape, is composed of several turns of wire surrounding an iron core of cylindrical shape, and carries about one ampere; this coil and core are enclosed in the fixed coil. The moving coil is suspended by a controlling spring which holds it in position at the top, and is supported by a small steel shaft at the bottom; this bottom bearing is so made that when the instrument is in use the moving system hangs freely from the controlling spring, and the vertical motion is so limited that excessive vibration cannot take place. Current is led to the

moving coil by two spiral conductors of negligible elasticity in comparison with the controlling spring; the controlling spring can be adjusted by changing its length.

The magnetomotive force in the fixed coil of an ammeter is about 2400 ampere-turns, and in the moving coil about 80 ampere-turns; with this combination, a torque of about 200 gramme-centimetres is obtained, which is from 80 to 200 times the torque of the ordinary measuring instrument in which the indications of a pointer on a scale are observed, and from 3 to 15 times the torque of the usual integrating, or curve-drawing instruments.

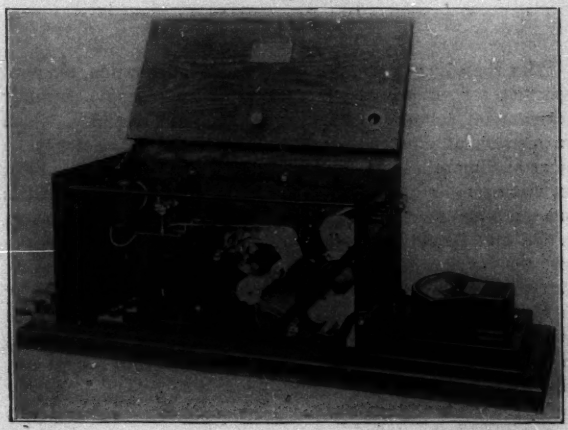

FIG. 1.

The pen used is a capillary tube supplied by a syphon which is filled from an ink reservoir placed at some distance from the recording point in the direction of the axis of the instrument. This arrangement secures a continual supply of ink and a fine line, and prevents blotting when the pen and paper are at rest. It also decreases the inertia of the moving system.

The different conditions under which the instrument is used require a variable pressure of the pen on the paper; if the instrument be used on a stationary platform, the pen should barely touch the paper; if on a car moving at high speed, the entire weight of the pen should rest on the paper. This adjustment

is made by means of a joint in the pointer and a small adjustable spring capable of supporting the entire weight of the movable part of the pointer.

The record is made on paper punched and ruled by a special machine and passed through the instrument by means of a toothed wheel driven by a spring motor, similar to that used for driving a phonograph. The speed of the paper can be adjusted to from four to eight inches per minute, but is ordinarily six inches per minute. The paper is about 3½ inches wide and is used in rolls 65 feet long. The machine that has been designed for ruling the paper can give any ruling, and special paper can be

FIG. 2.

made for each instrument; ordinarily, however, instruments of the same capacity are nearly enough alike to permit paper of uniform spacing to be used.

There is a second small pen employed to mark time accurately on the paper. This pen is actuated by a time-marker clock, through a small electromagnet, controlled by the clock. By this means, intervals of five seconds are marked on the paper. With this addition, it is possible to obtain simultaneous readings on a number of instruments placed upon different cars of a train. This marking device also serves to calibrate the speed at which

the paper is moving, thus making it unnecessary to calibrate the paper by means of a watch.

This construction is, of course, applicable to a voltmeter or wattmeter, as well as to an ammeter. By the use of these

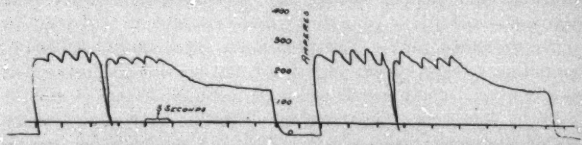


FIG. 3.—Current-curve taken by graphic recording-ammeter on a car equipped with multiple-unit control with automatic accelerating device.

instruments it is possible to obtain current and voltage readings with accuracy; the instrument does away with the labor of several men formerly required by the old method of two-second readings, and gives much more accurate results. It has become a simple matter to make all-day tests upon a railway motor, taking a sufficient number of readings to obtain a fair average, and thus determine the copper losses quickly and accurately. Knowing the voltage and current, it is possible to compute the core-loss of a motor from stand-tests and in this manner obtain the total energy-loss in heating the motor for any given service. The use of this instrument makes it comparatively simple in this way to determine the thermal capacity of a motor, and this with

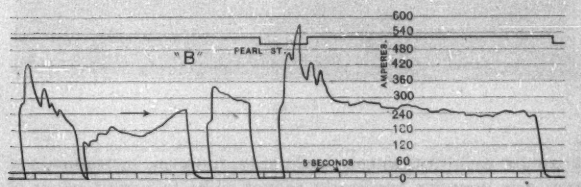


FIG. 4.—Current-curve taken by graphic recording-ammeter on one of the Schenectady-Albany cars equipped with four GE-73 motors. This car was climbing a heavy grade and the motors were in series.

such accuracy that consistent results are assured in comparing motors of different sizes and designs.

Figs. 1 and 2 give general views of the instrument, and Figs. 3, 4, 5, 6 and 7 copies of records taken under different conditions.

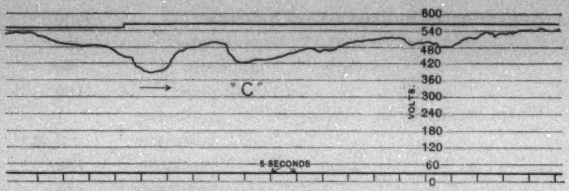


FIG. 5.—Voltage-curve taken by graphic recording-voltmeter on one of the Schenectady-Albany cars.

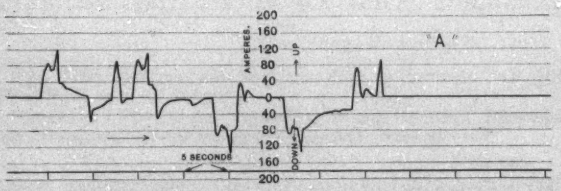


FIG. 6.—Current-curve taken by graphic recording-ammeter on an Otis elevator motor.

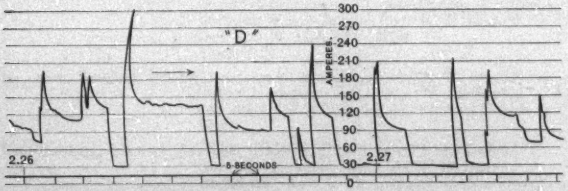


FIG. 7.—Current-curve taken by graphic recording-ammeter on a compound-wound generator.

The general data of the instrument shown in Figs. 1 and 2 is as follows:

Length, 32½″
Width, 13½″
Height, 11¼″
Approximate weight, 100 lbs.
Torque of ammeter, 210 gm-cm. (Full scale 24 degrees.)
" " voltmeter, 200 "
Period, 0.33 seconds for complete cycle.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ampere-turns in | fixed | coil | of | ammeter, | | | 2400 |
| " | " | " | " | " | voltmeter, | | 950 |
| " | " | moving | " | " | ammeter, | | 80 |
| " | " | " | " | " | voltmeter, | | 189 |
| Watts consumed in | moving | coil | of | ammeter, | | | 1 |
| " | " | " | " | " | voltmeter, | | 3.3 |
| " | " | fixed | " | " | ammeter, | | 130 |
| " | " | " | " | " | voltmeter. | | 33 |

## AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

NEW YORK, November 20, 1903.

The 181st Meeting was held this date at Carnegie Hall, 154 West Fifty-seventh Street.

SECRETARY POPE:—At a meeting of the Board of Directors this afternoon, the following 51 Associates were elected:

| Name. | Residence. | Endorsers. |
|---|---|---|
| AGNEW, JOHN PATTERSON, | Assistant Foreman Westinghouse E. & M. Co., Pittsburg; res., 755 Franklin Ave., Wilkinsburg, Pa. | C. F. Scott. P. M. Lincoln. C. E. Skinner. |
| BARNES, WALTER CLYMER, | Superintendent of Electrical Construction, Interborough Rapid Transit Co.; res., 200 W. 138th St., New York City. | H. A. Lardner. Calvin W. Rice. R. S. Masson. |
| BARNUM, THOMAS EDSON, | Assistant Chief Engineer, Cutler-Hammer Mfg. Co., 648 34th St., Milwaukee, Wis. | A. W. Berresford. W. L. Bliss. Samuel Sheldon. |
| BILDHAUSER, HENRY JEROME, | Electrical Engineer, General Electric Co., 44 Broad St., New York City. | F. C. Bates. J. B. Barr. C. H. Van Slyck. |
| COOLEY, FREDERICK EDMOND, | Testing Department, General Electric Co.; res., 785 State St., Schenectady, N. Y. | C. L. Cory. J. N. LeConte. G. C. Noble. |
| CRABB, CHARLES LOUIS, | Mechanical Engineer, Otis Elevator Co.; res., 171 Buena Vista Ave., Yonkers, N. Y. | C. B. Manville. F. W. Newell. J. D. Ihlder. |
| DAVIS, ERNEST EDGAR, | Mechanical and Electrical Engineer, Davis & Forest, Savannah, Ga. | C. E. Waddell. F. C. Caldwell. R. J. Nunn. |
| DE WOLF, ROGER DENNISON, | Electrical Engineer, Westinghouse Electric and Mfg. Co., Pittsburg; res., 508 Rebecca St., Wilkinsburg, Pa. | C. E. Skinner. N. W. Storer. F. C. Caldwell. |

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|---|---|---|
| **Esling, Albert,** | Sales Manager, R. E. T. Pringle Co., 18 Toronto Street, Toronto, Ont. | H. A. Moore.<br>R. J. Parke.<br>R. G. MacKeen. |
| **Field, Michael Birt,** | Contract Engineer, Ferranti Ltd.; res., 20 Cockspur St., London, S. W., Eng. | S. P. Thompson.<br>W. M. Mordey.<br>C. A. Carus-Wilson. |
| **Fitts, Clarence Dudley,** | Electrician, Oakville Co., Oakville, Conn. | J. W. Shuster.<br>B. V. Swenson.<br>W. H. Williams. |
| **Freudenberger, Lewis Alfred,** | Instructor, Delaware College, Newark, Del. | C. O. Mailloux.<br>W. S. Franklin.<br>William Esty. |
| **Goddard, Herbert Willard,** | Westinghouse, Church Kerr & Co., 10 Bridge St.; res., 176 W. 81st St., New York City. | A. D. O'Brien.<br>Horace Dowie.<br>F. S. Woodward. |
| **Gray, Roy William,** | Underground Foreman, Sunset T. & T. Co., 701½ South Broadway, Los Angeles, Cal. | L. E. Reynolds.<br>H. S. Clark.<br>C. A. Copeland. |
| **Groh, Bernard Charles,** | Engineer, Inter-State Telephone Co., of New Jersey, Trenton, N. J. | H. P. Clausen.<br>S. P. Grace.<br>L. W. Stanton. |
| **Gump, Walter Binkerd,** | Electrical Engineer, The Rumford Falls Power Co., Rumford Falls, Me. | A. L. Rice.<br>A. E. Kennelly.<br>A. A. Thresher. |
| **Hawkins, Charles Caesar,** | Electrical Engineer, H. Allen, Son & Co., Ltd.; res., 37 Conduit Road, Bedford, Eng. | S. P. Thompson.<br>James Swinburne.<br>R. W. Pope. |
| **Henry, David Carl,** | Chief Engineer, Henry Electric Equipment Co., 217 Grand Central Depot, Chicago, Ill. | T. C. Martin.<br>J. B. O'Hara.<br>W. D. Weaver. |
| **Hess, Adolf G. B.,** | 32 Seventh Ave., Brooklyn, N. Y. | W. J. Hammer.<br>C. P. Steinmetz.<br>W. D. Weaver. |
| **Hindert, Edwin George,** | Chief Mechanical and Electrical Engineer, The Cleveland and South Western Traction Co., 109 Wurst St., Elyria, Ohio. | E. O. Sessions.<br>E. B. Raymond.<br>J. W. Shuster. |
| **Hopkins, Robert Milne,** | Assistant C. A. Chapman, C. E., 1041 Marquette Bldg. Chicago; res., 621 Foster St., Evanston, Ill. | C. A. Dresser.<br>J. J. Gibson.<br>T. P. Gaylord. |
| **Hoor-Tempis, Maurice de.,** | Professor at the Royal University of Technical Sciences, Budapest II., Zsigmondutca 9, Hungary. | R. O. Heinrich.<br>Joseph Herzog.<br>James Swinburne. |
| **Hurd, Charles Henry,** | Consulting Engineer, Indianapolis and Chicago; res., 39 The Lexington, Indianapolis, Ind. | G. A. Damon.<br>R. W. Pope.<br>G. W. Nistle. |
| **Keeler, Irving Phelps,** | Superintendent of Power and Construction, Asheville Electric Co., Asheville, N. C. | C. E. Waddell.<br>A. M. Schoen.<br>Theodore Stebbins. |

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| KELLOGG, JAMES GIFFORD, | Student, Cornell University; res., 11 Central Ave., Ithaca, N. Y. | H. J. Ryan.<br>G. S. Macomber.<br>R. W. Pope. |
| KIMBALL, ROGER NELSON, | Vice-President and Superintendent, Kenosha Gas and Electric Co., 210 Wisconsin St., Kenosha, Wis. | J. J. Gibson.<br>E. B. Clark.<br>B. J. Arnold. |
| KNOTT, HENDERSON WILLIAM, | Engineer, Mohawk Electric Mfg. Co., 15 Cortlandt St., New York City. | Samuel Sheldon.<br>T. C. Martin.<br>R. W. Pope. |
| KUNZE, RUDOLPH I., | Electrical Engineer, Otis Elevator Co.; res., 17 Culver St., Yonkers, N. Y. | C. B. Manville.<br>F. W. Newell.<br>F. L. Skidmore. |
| LANDERS, GEORGE FOREMAN, | Captain Artillery Corps and in charge of Department Submarine Defense, Fort Totten, N. Y. | Samuel Reber.<br>G. O. Squier.<br>Edgar Russel. |
| LEWIS, EMANUEL WILLIAM, | Special Apprentice, Westinghouse Elec. and Mfg. Co.; res., 5313 Broad St., Pittsburg, Pa. | C. E. Downton.<br>H. P. Gibbs.<br>F. J. Chisholm |
| MAXWELL, ALEXANDER, | Assistant Superintendent 3d District, New York Edison Co.; res., 6 East 126th St., New York City. | J. W. Lieb, Jr.<br>W. F. Wells.<br>E. A. Leslie. |
| MCLEARY, SAMUEL HARVEY, | Student Cornell University; res., 325 Eddy St., Ithaca, N. Y. | H. J. Ryan.<br>H. H. Norris.<br>G. S. Macomber. |
| MEARSON, FRANCIS; | Solicitor and Estimator, Geneva Electric Co.; res., 41 W. 114th St., New York City. | E. B. Benjamin.<br>S. W. Ashe.<br>William Mackay. |
| MILLER, THOMAS LODWICK, | Partner Miller & Wilson, 7 Tower Buildings, Water St. res., 7 Lancaster Ave., Lipton Park, Liverpool, Eng. | C. H. Wordingham<br>W. H. Donner.<br>Philip Dawson. |
| MÖLLER, JACOB ALARIC LEO, | Engineer, Lambert Schmidt Telephone Mfg. Co., Inc., 85 Maple St., Hoboken, N. J.; res., 63 Riverside Drive, N. Y. City. | Townsend Wolcott.<br>Lambert Schmidt.<br>W. T. Booth. |
| MOWBRAY, WILLIAM J., | Engineer, Brooklyn Edison Co., 14 Rockwell Pl.; res., 272 Classon Ave., Brooklyn, N. Y. | Samuel Sheldon.<br>E. A. Leslie.<br>W. F. Wells. |
| OBEAR, GEORGE BARROWS, | Instructor in Electric Engineering, Lowell Textile School, Lowell; res., 640 Western Ave., Lynn, Mass. | Louis Duncan.<br>C. R. Cross.<br>C.J.H.Woodbury. |
| OSWALD, HERMAN HENRY, | Student General Electric Co.; res., 306 Lafayette St., Schenectady, N. Y. | H. F. Hoxie.<br>A. L. Rohrer.<br>E. B. Raymond. |
| ROWE, BERTRAND PERRY, | Detail Engineering Office, Westinghouse Electric and Mfg. Co., Pittsburg; res., 911 Ramsey Ave., Wilkinsburg. | C. F. Scott.<br>P. M. Lincoln.<br>H. P. Davis. |

| | | |
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| SAWYER, BURTON MANSFIELD, | Electrical Engineer, Westinghouse Electric and Mfg. Co.; res., 5811 Rippey St., Pittsburg, Pa. | J. S. Peck. C. E. Skinner. E. M. Tingley. |
| SCHMITT, FREDERICK E., | Associate Editor, Engineering News, 220 Broadway, New York City. | W. H. Browne, Jr. E. H. Mullin. C. F. Burgess. |
| SIEGFRIED, JOSEPH HENRY, | Union Electric Light & Power Co., St. Louis, Mo. | H. W. Leitch. J. P. Jackson. W. F. White. |
| SLOCUM, ARTHUR GAYLORD, JR., | Operator, Central Power Station, St. Louis Transit Co.; res., 3426 Washington Ave., St. Louis, Mo. | H. C. Hoyt. C. L. Stone. G. Lindman. |
| SMITH, JULIAN CLEVELAND, | Superintendent, The Shawinigan Water and Power Co.; res., 1724 Notre Dame St., Montreal, Que. | J. A. Walls. R. W. Pope. W. C. Johnson. |
| STAFFORD, REX THOMAS, | Electrical Construction Foreman, Lackawanna Steel Co.; res., 126 Cottage St., Buffalo, N. Y. | C. E. Gifford. C. R. Huntley. E. D. Edmonston. |
| STEVENS, JOSEPH BROWN, | Assistant Foreman Motor Testing Department, General Electric Co.; res., Cleveland Ave., West Lynn, Mass. | G. E. Sanford. E. E. Boyer. C. B. Burleigh. |
| TURNER, HARRY WINTHROP, | Winding and Insulation Specialist, British Thomson Houston Electric Co., Ltd.; res., 37 Hillmorton Road, Rugby, Eng. | Calvin W. Rice. H. M. Hobart. C. B. Burleigh. |
| WALTER, HARRY CASPER, | Instructor, Purdue University; res., 143 Andrew Pl., West Lafayette, Ind. | J. W. Esterline. W. E. Goldsborough. H. B. Smith. |
| WHITTLESEY, JAMES THOMAS, | Chief Engineer, Elec. Dept., Public Service Corporation; res., 39 Lloyd Pl., Montclair, N. J. | Frederick Bedell. Paul Spencer. W. S. Barstow. |
| WILLIS, SAMUEL THAYER, | Switchboard Inspector, Stanley Electric Mfg. Co.; res., 16 Melville Blk., Pittsfield, Mass. | C. C. Chesney. W. J. Lloyd. Gilbert Wright. |
| WOHLAUER, ALFRED, | Student, Testing Dept., General Electric Co.; res., 211 Liberty St., Schenectady, N. Y. | R. N. Williams. E. B. Raymond. J. E. Noeggerath. |

Total, 51.

The following were transferred from Associate to the grade of Member.

WALTER AMBROSE PEARSON, Electrical Engineer, Interurban Street Railway Co., 621 Broadway, New York City.

WYNN MEREDITH, Designer and Inventor, San Francisco, Cal.

LORIN EVERETT IMLAY, Asst. Supt., Niagara Falls Power Co., Niagara Falls, N. Y.

BERNARD EDWARD SUNNY, Western Manager General Electric Co., 1047 Monadnock Building, Chicago, Ill.

JOHN JOSEPH CARTY, Chief Engineer New York Telephone Company, 18 Cortlandt St., New York City.

EUGENE B. CLARK, Electrical Engineer to Illinois Steel Co., Chicago, Ill.

*A paper presented at the 181st Meeting of the American Institute of Electrical Engineers, New York, Nov. 20, 1903.*
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# THE ELECTRICAL CONDUCTIVITY OF COMMERCIAL COPPER.

BY LAWRENCE ADDICKS.

Electrical conductivity has come to be a universally accepted measure of the degree of purity of copper for two reasons: First, on account of the enormous quantities of copper used in electrical construction; Secondly, because the conductivity forms a very delicate test of the chemical purity of the metal. Furthermore, a conductivity test may be made upon a wire of suitable dimensions in about three minutes, while a chemical analysis of refined copper is a tedious and correspondingly expensive operation.

Matthiessen's standard (one metre-gramme of pure soft copper at 0° Centigrade = 0.14172 international ohms), has come into very general use, although it is some three percent too low. The fact that the quality of the copper on the market to-day often exceeds by as much as one percent the purest laboratory copper of forty years ago forms an interesting commentary on the excellence of modern methods.

Conductivity is affected by the physical state of a metal or alloy as well as by its chemical constitution. Now in order to obtain the sample of wire upon which a measurement is made, the copper under examination must be subjected to several mechanical processes, and these different variables must be taken into account. It is the purpose of this paper to point out their influence.

First, let us take up the influence of the chemical impurities commonly met with. These consist of oxygen in the form of suboxid of copper, arsenic, antimony, and, in some coppers, bismuth. Small quantities of some other elements, may be met with but those mentioned are the impurities to be looked for in low copper.

The relation between arsenic or antimony and conductivity is shown in Fig. 1. As arsenic and antimony have a very similar influence the data have been gathered as arsenic plus

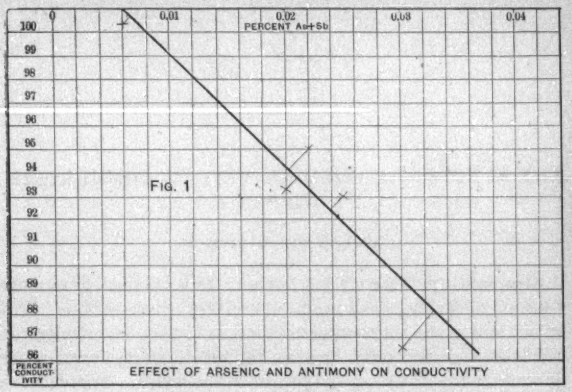


FIG. 1

EFFECT OF ARSENIC AND ANTIMONY ON CONDUCTIVITY

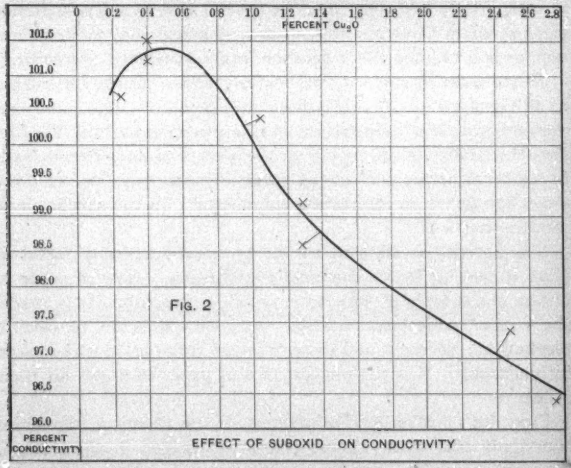


FIG. 2

EFFECT OF SUBOXID ON CONDUCTIVITY

antimony, thereby greatly simplifying the work. The suboxid of copper is practically constant in all these determinations. The conductivity is stated for annealed samples of wire. It will be

noticed that 1/100 percent of arsenic or antimony lowers the conductivity about five percent.

Fig. 2 shows the relation between suboxid and conductivity in annealed wire. The suboxid has been calculated from the oxygen, itself obtained by subtracting the percent copper by analysis from 100. The quantity of other impurities present is so small that this is substantially correct. The crest of the curve shows copper " in pitch " or " set " or when suffi-

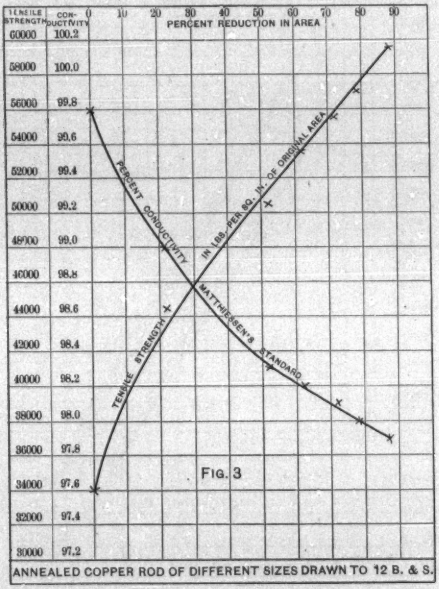


FIG. 3

cient oxygen is retained to keep the impurities as oxids, in which state they are supposed to be less harmful. The possible formation of a carbid or the absorption of hydrocarbon gases from " overpoling " in the refining furnace has also been suggested.

We come now to the physical characteristics and their influence upon the conductivity. We can cause considerable changes in structure by mechanical operations and, as would be expected, the conductivity is affected thereby, but by no means as seriously

as by arsenic or antimony. The operation most commonly met with is that of wire drawing—conductivity varies with hardness. In Fig. 3 conductivity and tensile strength are plotted against the percentage reduction in area of a wire rod after annealing and in Fig. 4 conductivity is plotted directly with tensile strength. It will be seen that the tensile strength varies almost directly as the reduction in area and that the conductivity drops about 1/10 percent for every 1,300 lb.

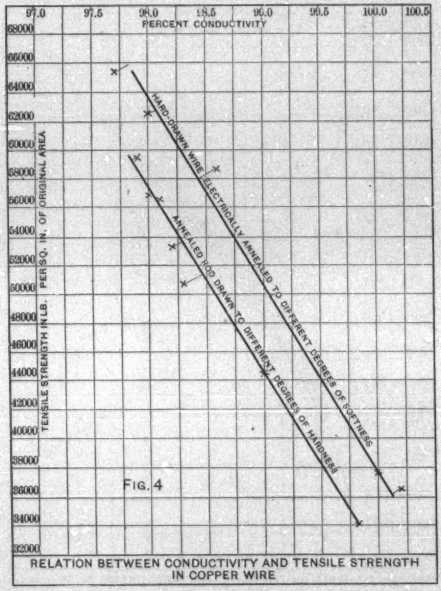


FIG. 4

RELATION BETWEEN CONDUCTIVITY AND TENSILE STRENGTH IN COPPER WIRE

gain in tensile strength, a fact which must be taken into account when comparing different samples of hard-drawn wire. Fig. 4 also shows results obtained where a hard-drawn wire is given different degrees of annealing. This ratio, shown by the angle of inclination of the straight line to the axes, is independent, in so far as the writer's investigations have extended, of the actual conductivity of the annealed wire, which is of course dependent upon chemical purity.

Fig. 5 shows the effect of temperature in the annealing operation. A hard-drawn, 12 B. & S., wire has been heated to various temperatures by being forced to carry currents from zero to a fusing current of about 220 amperes. The actual temperatures were not measured at the time the tests were made. Very roughly the temperature in degrees Fahrenheit will be about ten times the values shown for amperes. In the experiments quoted about ten feet of wire was put in series with a water rheostat and the whole shunted around forty or fifty volts of a 4,000-ampere circuit. The current was adjusted by acidifying the water in the rheostat and was left on in each case until it

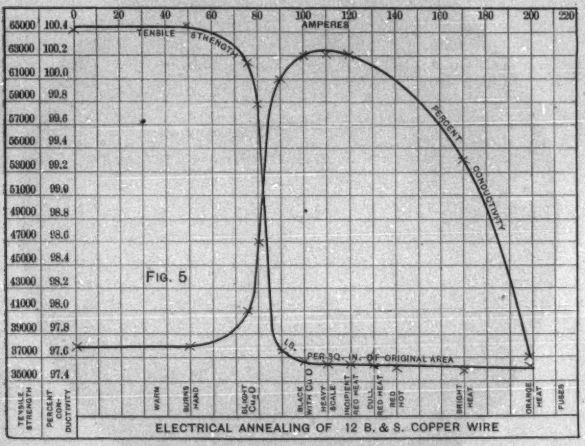

FIG. 5

reached a steady value—a matter of about a minute. The time during which the current was on, after this steady heat was reached, seemed to exert no other effect than to increase the shell of scale formed on the wire.

The question at once arises whether the lower conductivity due to overheating is caused by the absorption of oxygen in some way or to a crystalline change. The first idea that suggested itself was to anneal the wire in such a way that oxygen could not attack it. Handling white-hot wire of the length needed in an atmosphere of nitrogen proved somewhat troublesome, however, and was left as a last resort. Silver-plated wire was tried but the plating would not stand the temperature Finally the

problem was attacked in two different ways: First, samples of burned wire were immersed in dilute nitric acid until the diameter was considerably reduced and then measured for conductivity. No appreciable difference was found, showing that any chemical change was at least uniform throughout the wire; Secondly, samples of burned wire were drawn to a smaller size, re-annealed properly and then tested. The original wire, when properly annealed showed 100.4 percent conductivity; when burned for the test, 99.1 percent; and when drawn down and re-annealed

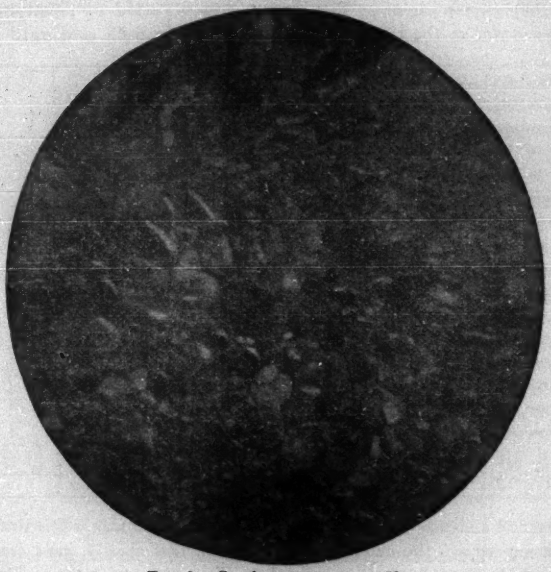

FIG. 6.—Good cast copper—×50.

properly, 100.5 percent. These results show the change to be of a physical, not chemical nature.

Figs. 6, 7 and 8 are micro-photographs of copper samples which have been polished and etched in 50-percent aqua-regia for about 30 seconds. The magnification and the size of rod from which the samples have been cut are the same in each case. Fig. 6 shows good copper in "set", having a conductivity when drawn and annealed of 99.7 percent. Pure metals, when cast, crystallize on cooling into irregular grains the size and distribution of which

depend largely upon the size of casting and rate of cooling. When small quantities of impurities are present they form a matrix of what is probably an eutectic alloy of the metal and the impurities, in which the grains of pure metal are set. This is the case with the sample under discussion where the total impurities amount to about one-tenth percent. Each of these grains is made up of smaller crystals, the cleavage being in different directions in grains. Fig. 7 shows a copper low enough in pitch to give trouble at the draw-bench. It is the lowest of the

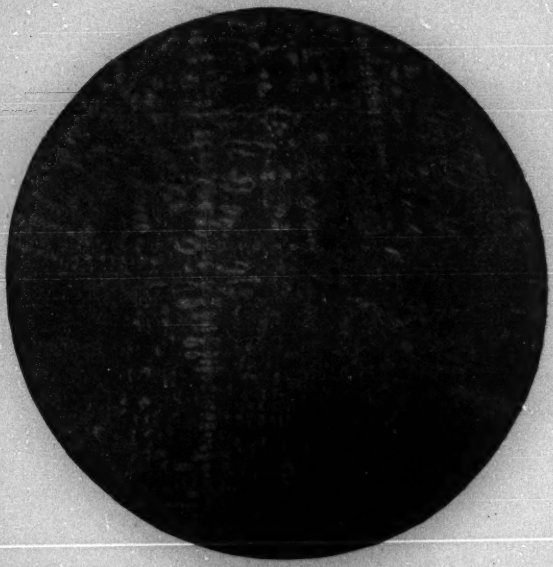

FIG. 7.—"Low" cast copper—×50.

series of points on Fig. 2 and runs 96.4 percent annealed. The change in structure is at once apparent and accounts for the brittleness developed.

When copper is rolled and drawn this granular structure is broken down, a fine fibre resulting. Fig. 8 shows a sample of trolley wire which has been heated over a gas-blast to a bright heat. It shows a partial return to the crystalline state, the matte surface in places having the same appearance as a hard-drawn sample as far as fibre is concerned. This explains the lowering

of conductivity by overheating. The writer has never made measurements of the conductivity of cast copper, but understands from work done at one of the wire-mills that it has been found to run about 3.5 percent lower than the annealed wire. This is entirely in accord with the data shown above. Copper that has been re-melted in the foundry may run but 30 or 40 percent, due to the absorption of gases.

In cast copper we have a conductivity resulting from the combined conductivities of the pure copper grains and the impure

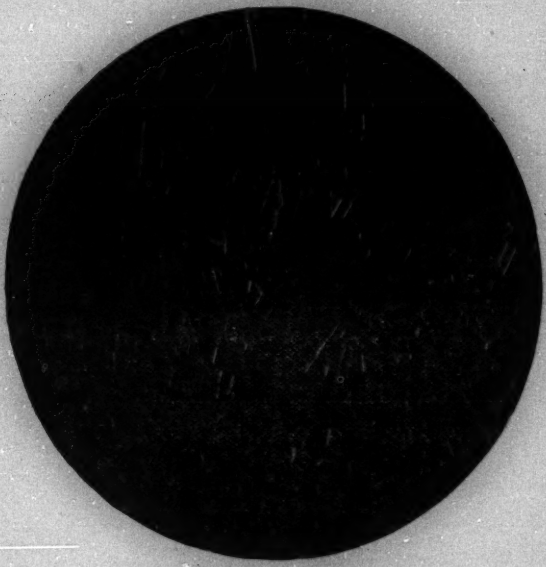

FIG. 8.—"Burned" trolley wire—×50.

matrix or cement. When this is rolled the structure is broken down and the complex shunt and series circuits rearranged to better advantage. When hard-drawn, the molecular freedom is restrained and more work has to be done in the necessary oscillation of an atom in passing a charge on to its neighbor. In annealing, the wire is brought to a temperature at which the cement is softened and all internal strain is equalized. In burning, crystalline growth is started and reverts the metal to the original condition.

## Discussion on Mr. Addick's Paper.

The PRESIDENT:—The paper just read gives much interesting information in regard to the properties of copper wire. Practically the only specification that engineers are able to use is that copper shall have a conductivity 98 percent of that of pure copper; all else is left to the manufacturer. If the author had embodied a specification for copper for railroad and lighting work, it would have been most beneficial.

F. J. NEWBURY:—The results given in this paper are confirmed by our work. Our test for the purity of copper is a conductivity test. Manufacturers of copper wire would greatly prefer that the specification should apply to the strands making up a wire, and not to the stranded wire as a whole; the resistance of the stranded wire is greater than that of the individual wires composing it, and the strength of the stranded wire is less. The difference, however, between the stranded wire and its component parts is not always the same; the component wires, if hard drawn, should have a tensile strength of 60,000 pounds per square inch, and a conductivity of 97 percent Matthiessen standard. The resistance of stranded wire will be approximately 3 percent greater, and this seems to be apparently the amount taken up in the twist of the wires, which indicates that the current follows the individual wires around the spiral. Commercial copper, as now made, is very nearly equal to the conductivity shown by the tables published by the INSTITUTE; in strength it varies from 52,000 to 65,000 pounds per square inch; 68,000 pounds is the maximum, and 60,000 for an ordinary specification is fair.

The PRESIDENT:—Engineers will probably not agree with Mr. Newbury in his preference for a specification of the component wires of a stranded wire; the engineer wishes to know the result to be expected from the wire, and cares little for the factors entering into it; he does not wish to have to make allowance for the manner in which the wire is made up. Possibly the matter of heating enters into the consideration of a stranded cable; a bare conductor will carry more energy than an insulated conductor. Is it not possible that the twisting of the wires reduces the conductivity a portion of that 3 percent?

*A paper presented at the 181st Meeting of the American Institute of Electrical Engineers, New York, Nov. 20, 1903*
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# THE COMPARATIVE BEHAVIOR OF FLOATING AND BOOSTER-CONTROLLED BATTERIES ON FLUCTUATING LOADS.

BY LAMAR LYNDON.

*Methods of Application.*—The object of using the storage-battery as a regulator on variable loads is to absorb, as far as possible, all fluctuations above or below the average current. When the external load is light, the generator sends current into the battery as well as to the external circuit. When the demand is heavy, the battery discharges in parallel with the generator, relieving the latter of the excess load. In some instances it is desired to maintain the generator current constant, eliminating the shock and strain of excessive and suddenly-imposed loads, and to secure the fuel-economy incident to steady loads as compared with fluctuating. In other cases, the object is to maintain a constant voltage on a circuit or rather to prevent it from varying more than a predetermined amount. Obviously, the maintenance of absolutely constant load, or voltage, is the ideal condition, and the more nearly this is approached the more nearly is the object attained.

Where the load is applied some distance from the power-station, there are five possible methods of installing a storage-battery to maintain the load on the generator reasonably constant, which are:

(1) Floating battery in power-station.

(2) Floating battery, out on line, at point of application of load.

(3) Battery with booster, in power-station.

(4) Battery and booster, out on line, at point of application of the load.

(5) Battery out on line, at point of application of the load, and booster in the power-station.

*Cell Characteristic.*—Fig. (1) shows the characteristic of an 800-ampere-hour cell one-quarter discharged, at 70° F., in good condition. This will change with temperature, the characteristic more nearly approaching the horizontal with higher, and the perpendicular with lower, temperature. It will also change with state of charge, density of electrolyte, and condition of plates. The ordinates above the horizontal line which intersects the characteristic curve at zero represent the voltage of a cell on charge, and those below this line the volts on discharge, at various rates of current flow. The abscissas are amperes, the curve being extended on either side of the zero to that point where the current flow is equal to the one-hour rate of charge or discharge. Each reading was taken 20 seconds after beginning of the corresponding current

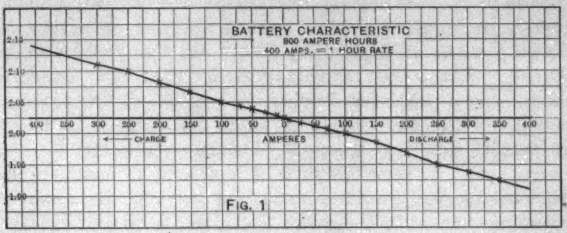



FIG. 1

flow. The zero point shows the voltage of a cell when current flows neither into nor out of it.

This is the general form of characteristic for nearly any type or size of cell, the conditions of charge, temperature, density of electrolyte, etc., being the same. The data then to construct the characteristic for any size of battery of same state of charge, temperature, and density of electrolyte as given are:

Volts per cell at zero, 2.025; at the one-hour discharge rate 1.915, and at the one-hour charge rate 2.14. Connect these points by a straight line. The one shown is practically a straight line, and repeated tests on many sizes and types of cells indicate that if the conditions be maintained constant throughout a test, and due allowance be made for the error in reading the small changes in the voltmeter deflections, the characteristic is a straight line between the points of one-hour charge and one-hour discharge rates.

*Conditions Necessary for Floating.*—If a storage-battery be "floated" on a system on which the load is fluctuating, the voltage of the circuit across which it is connected must obviously vary with the load; rising above that of the battery when the demand is light and sending in a charging current; falling when the demand is heavy and allowing the battery to discharge and assist the generator.

In order that the system may work properly there should be a certain relation between the battery characteristic and that of the generator. The greater the inclination of the generator characteristic to the horizontal; that is, the greater the change in

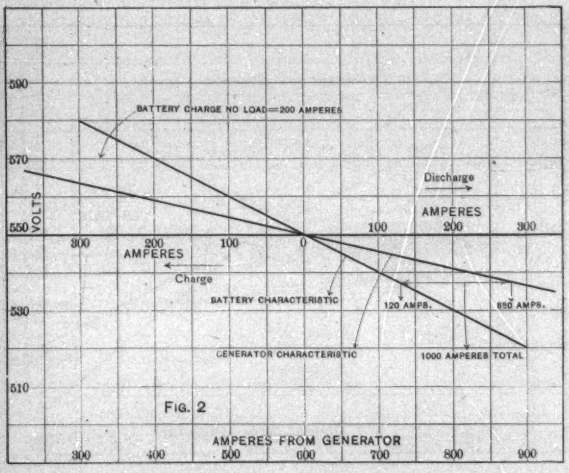



Fig. 2

voltage with change in load, the greater will be the battery discharge on loads in excess of the normal or average load. Referring to Figs. 2 and 3. In Fig. 2 the generator characteristic has a certain inclination to the horizontal. The battery characteristic is superimposed on it, the point of zero current flow intersecting the generator curve at the point of normal load—600 amperes. For simplicity both are here assumed to be straight lines. When 1,000 amperes are sent over the line, the generator furnishes 880 and the battery 120 amperes, the increase in load on the generator being 280 amperes or 47 percent.

In Fig. 3 the inclination of the generator characteristic is greater than in Fig. 2, and for 1,000 amperes station output, the generator furnishes 750 amperes, the battery 250, the increase of generator load above normal being 150 amperes or 25 percent. In the latter case, however, on light load, the voltage rises so greatly that an excessive current will pass into the battery, probably injuring it. As the one-hour rate of charge should never be exceeded—and even this should last but a very short time—the relative inclination of the two characteristics must come within certain limits. Furthermore, large shunt generators of high efficiency cannot be made—or at least certainly are not made—so that the full-load voltage decreases more than 15

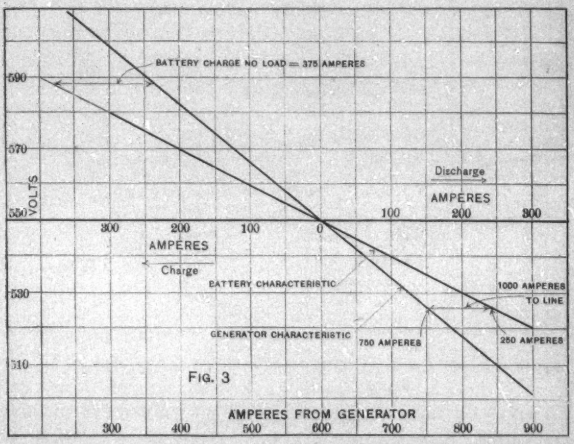



FIG. 3

percent below the no-load voltage. This limits the inclination of the generator characteristic.

CASE I.—FLOATING BATTERY IN STATION.

*Data of System.*—Consider the case of a fluctuating load such as is shown in Curve A, Fig. 7. The minimum current is 260, maximum 1,000, and average 600 amperes. Such a load coming directly on the generating equipment must result in low efficiency and high fuel-consumption per h.p. hour; and will require generators with large overload capacity and heavy engines to withstand the strains imposed. It may here be mentioned that the load-curve shown is not an assumed one, but represents the actual readings made on an operating electric railway.

Assume that this load is carried over one feeder to a point 2 miles from the power-station; that the minimum voltage at any time is to be 350 volts; that the track is of 70 lb. rail, well bonded. Still assuming both characteristics as straight lines, Fig. 4 is drawn. Line "I" is the generator characteristic. The dotted line running just underneath "I" is the characteristic of a 600-ampere-hour battery, the one-hour charge or discharge rate of which is 300 amperes. This line is in reality coincident with the generator curve. The voltage of the generator at normal load, 600 amperes, is 550 volts, which is also the open-circuit voltage of the battery. The ordinates are volts as indicated. The lower set of abscissas are amperes of generator output; the set on the hori-

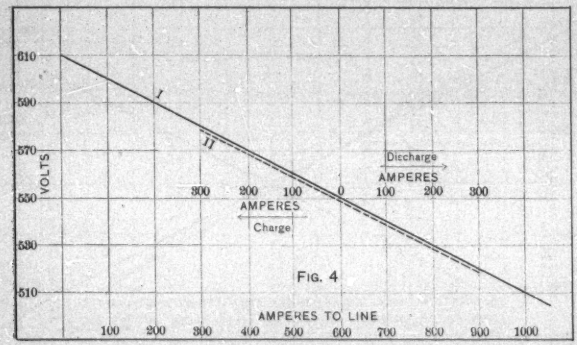



FIG. 4

zontal line at 550 volts are amperes of charge or discharge of the battery.

*Summation Curve.*—Fig. 5 shows a summation curve which gives the station voltage at any output to the external circuit and is constructed as follows:

At voltage of 530 the generator output is 800 amperes, the battery discharge 200; total, 1,000 amperes to the line at this voltage. Similarly, at 540 volts the sum of the two outputs = 700 + 100 = 800; at 560 volts the generator output = 500 amperes and the current going into the battery = 100 amperes; hence the line current = 500 − 100 = 400. Likewise, at 570 volts, line current = 400 − 200 = 200 amperes. In this way the points on the summation curve are located which give line

current as the abscissas and station voltage as the ordinates. Since the characteristics are both straight lines, the summation curve is also a straight line. In this case only two points have to be located and joined together, but in many cases where the generator characteristic is a curve, the summation line becomes a curve and several points must be located to trace it. For this reason the method of tracing is here set forth.

*Line Constants.*—From Fig. 5 it may be seen that the station voltage at 1,000 amperes is 530 volts. Since the minimum voltage at the delivery point 2 miles distant must not fall below 350 volts, the total line-drop must not exceed 180 volts, which fixes the resistance at .18 ohm. The resistance of two miles

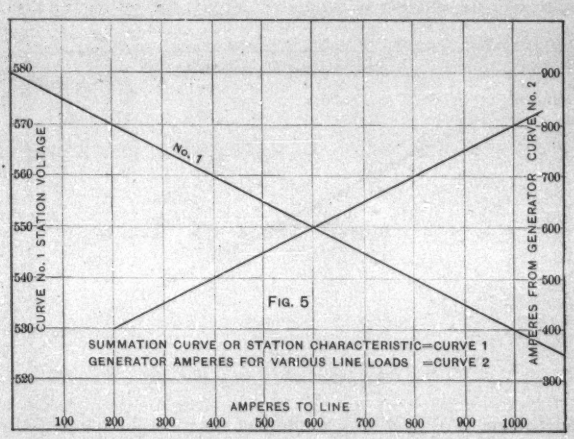



FIG. 5

of well-bonded 70 lb. rails is .0582 ohm. This leaves .1218 ohm as the copper resistance or .0609 ohm per mile, corresponding to 940,000 cir. mils. Taking this resistance, the curve of line-drop at various loads is plotted, which is shown in Fig. 6. This is, of course, a straight line.

*Derived Diagrams.*—From the foregoing data and figures the curves in Fig. 7 are derived. Curve A is the load-diagram before mentioned; Curve B, the generator load; Curve C, the current to or from the battery; Curve D, the variation in station voltage with load, and curve E, the change of voltage at the feeding point, plotted by subtracting from the ordinates

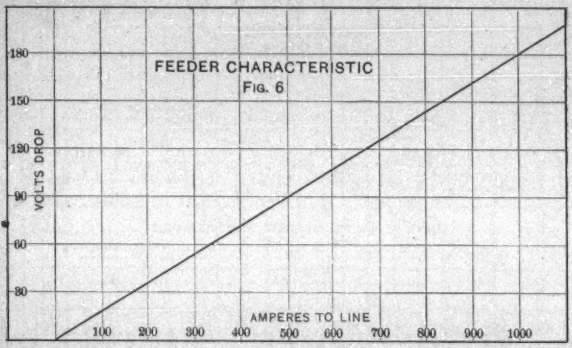
of D the feeder-drop for the various currents. Since all the relations are of the first order, the various curves are of the same form as the load-curve. Curves D and E are also of the same form, but reversed, being inverse functions of the load-curve.

A study of these curves reveals the following:

(1) The voltage at the station falls off rapidly with increase of load (Curve D, Fig. 7).

(2) Only two thirds of the capacity of the battery to discharge are utilized. It can discharge at the rate of 300 amperes, and only 200 amperes are drawn from it at the instant of the maximum peak (Fig. 4).

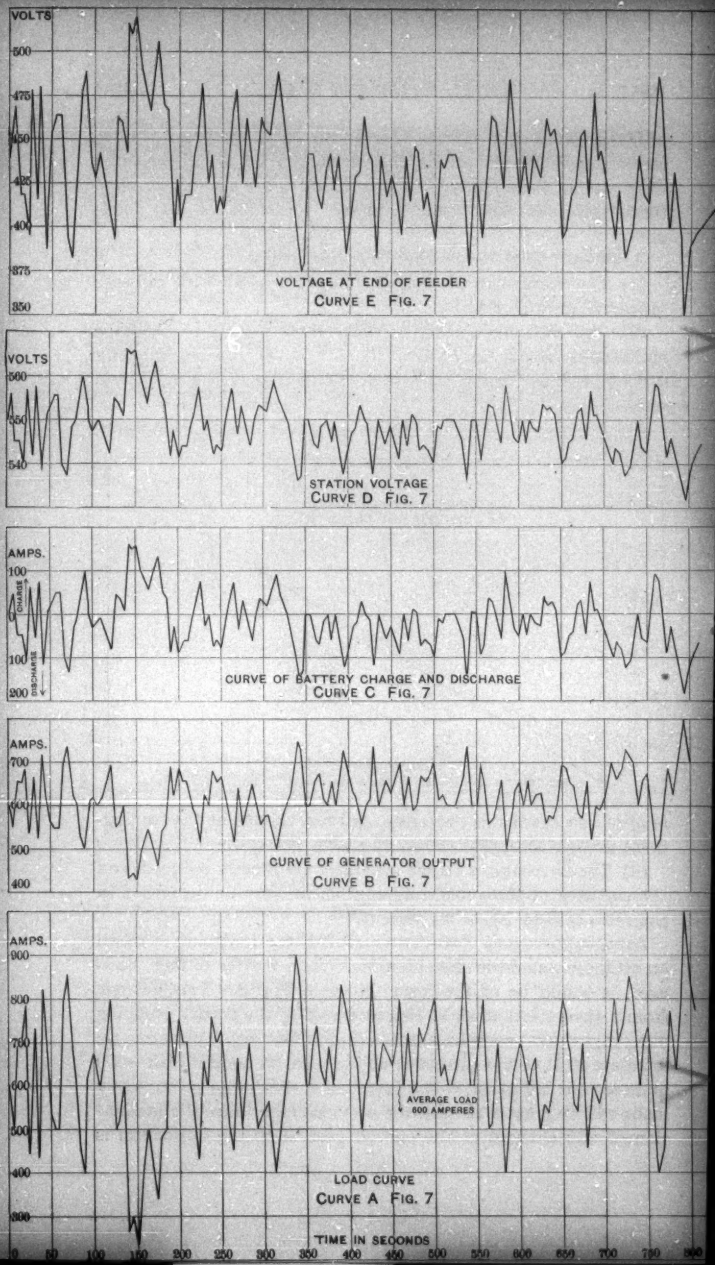
(3) Because of the fall in station voltage with load, the copper




required on the line is excessive, and the voltage at the feeding-point unsatisfactory, as shown by Curve E.

(4) The fluctuations on the generator are greatly reduced, and the capacity of the generating equipment may be decreased in the ratio of 8 to 10, or 20 percent.

*Drooping Generator Characteristic.*—All the foregoing is based on straight generator characteristic. As a matter of fact, however, it would be of the form shown in Fig. 8. This change, from a theoretical straight characteristic to the actual drooping one, introduces complications. If the battery characteristic intersect that of the generator at the point of normal load—600 amperes—as indicated by the dotted line, the discharge on heavy loads will be greater than in the previous case, since the inclination of any element of the curved portion to the horizontal is


VOLTS
500
475
450
425
400
375
350
VOLTAGE AT END OF FEEDER
CURVE E FIG. 7
VOLTS
560
550
540
STATION VOLTAGE
CURVE D FIG. 7
AMPS.
100
CHARGE
DISCHARGE
200
CURVE OF BATTERY CHARGE AND DISCHARGE
CURVE C FIG. 7
AMPS.
700
600
500
400
CURVE OF GENERATOR OUTPUT
CURVE B FIG. 7
AMPS.
900
800
700
600
500
400
300
AVERAGE LOAD
600 AMPERES
LOAD CURVE
CURVE A FIG. 7
TIME IN SECONDS
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800

greater than that of the straight-line characteristic, and a larger portion of the load will be taken by the battery. But it may also be seen that with the same light loads as before, the battery-charging current is less than in the previous case.

*Relation between Input and Output.*—In any battery used for regulation, it is necessary that the input approximately equal the output, otherwise the state of battery charge is changed and the efficiency of the regulation decreased. If the input exceed the output, the excess energy will be lost, going off in the form of free gases from the cells, and the high voltage due to overcharging decreases the current which may be sent into the battery on light load, thus permitting more fluctuation of the generator load. Furthermore, continuous overcharge will ruin a battery, if persisted in.

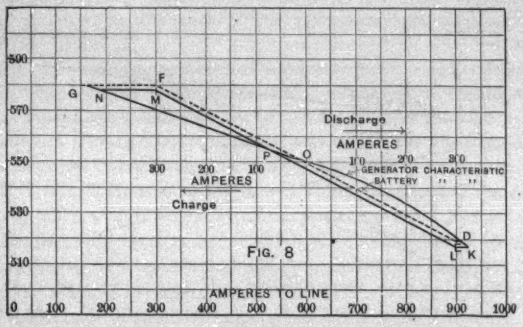



FIG. 8

If the output exceed the input, electrical bankruptcy follows and the battery is in no condition to help the generator when most needed.

From Fig. 8 it may be seen that the input and output are to each other inversely as the two areas on either side of the intersection of the generator and battery characteristics, which are formed by the spaces included between the two characteristics and closed by horizontal lines which are drawn through the points of voltage at maximum charge and discharge. That is, the areas O D E and O F G are to each other inversely as the battery input is to the output. The point O is not necessarily the intersection with the line of average external load, but is the point of intersection of the two characteristics. Obviously, therefore, the battery characteristic must be moved downwards,

increasing O F G and diminishing O D E until these two areas are equal, the resulting areas being P M N and P L K. This means that the normal battery voltage is lowered, its zero point being shifted downwards. With the normal load of 600 amperes on the line, there will now be a charging current into the battery of 20 amperes, making the generator current 620 amperes and lowering the station voltage to 548 volts.

*Conditions with Drooping Characteristic.*—The conditions are then manifestly less advantageous in the actual case than in the theoretical case first discussed and in either case are worse than they would be with an over-compounded generator in the power-station working without a battery. The mere reduction of mechanical strains and fuel-consumption are of little avail if the

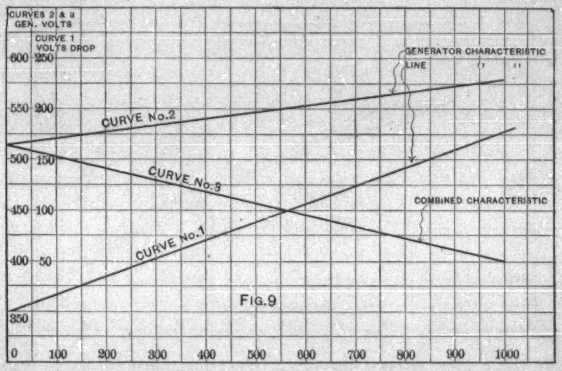



FIG. 9

cars on the system cannot be accelerated, carried over grades and schedules maintained.

*Compound Generator.*—For comparison with the previous cases, Figs. 9 and 10 are shown. In Fig. 9 are plotted the characteristic of an over-compounded generator, the line characteristic with resistance .18 ohms as before and a combination of these showing the total characteristic for the feeding-point. From this last, the curve in Fig. 10, which is the voltage curve at the feeding-point, is plotted. This is drawn to the same scale as Curve " E," Fig. 7, and a comparison of these two curves shows the superiority of the over-compounded generator working alone. The minimum voltage in the former case is 350, in the latter 401; and the maxima are 522 and 482 volts respectively. If 350 volts

are sufficient for satisfactory operation, the resistance of the circuit may be increased from .18 to .23 ohm. Deducting the track resistance, .0582 ohm, the copper resistance becomes .1718 ohm which corresponds to 665,000 cir. mils, as against 946,000 cir. mils. in the first case—a reduction of 29 percent.

CASE II.—FLOATING BATTERY OUT ON THE LINE.

*Data of System.*—Consider now the case of a floating battery located at the feed-point—2 miles away from the station: Assume an over-compounded generator giving 520 volts at no load, and 580 volts at full load of 800 amperes, and assume further that the battery will discharge 200 amperes at maximum load of 1,000 amperes, giving 800 amperes as the maximum transmitted over the line. If the minimum voltage allowable be 350 volts, the resistance = .287 ohm or .2288 ohm for the copper resistance, corresponding to 495,000 cir. mils. or 25 percent less copper than that required in the case of the compound-wound generator working alone.

Fig. 11 shows the generator characteristic, the line characteristic, the combined characteristic, and the characteristic of a 600-ampere-hour, 392-volt battery. This shows that when the maximum load of 1,000 amperes comes on the system the generator supplies 700 amperes, the battery 300, and the voltage is 371.

*Derived Diagrams.*—In Fig. 12 are shown the summation curve of Fig. 11, giving the voltage at the feeder-point for any current to the line, and the line No. 2 which shows the relation between the amperes flowing over the line and the current from the generator. Fig. 13 shows the three curves: B the load on the generator; C the current to or from the battery, and D the voltage at the feed-point all for the same load curve shown in A, Fig. 7. All these are to the same scale as the corresponding curves in Fig. 7 and may be readily compared therewith.

*Excessive Rate of Charge.*—The only objectionable feature in this system is, that the charging current, if the load on the line should reduce to zero, would be far in excess of the permissible rate. This is shown in both Figs. 11 and 12. In Fig. 12 the voltage at zero line-load is seen to be 425 volts, which on the combined characteristic in Fig. 11 corresponds to a current-flow over the feeder of 450 amperes. If there is any possibility of the line ever being relieved of load, some provision must be made for avoiding this excessive charge. In the system under discussion, however, this never occurs.

*Results.*—The location of a floating battery at the feeding-point shows manifest and undoubted improvement in that

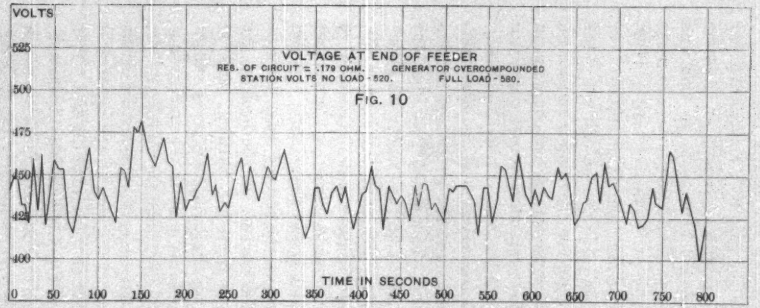

VOLTS
525
500
475
450
425
400
VOLTAGE AT END OF FEEDER
RES. OF CIRCUIT = .179 OHM. GENERATOR OVERCOMPOUNDED
STATION VOLTS NO LOAD - 520. FULL LOAD - 580.
TIME IN SECONDS
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800

FIG. 10

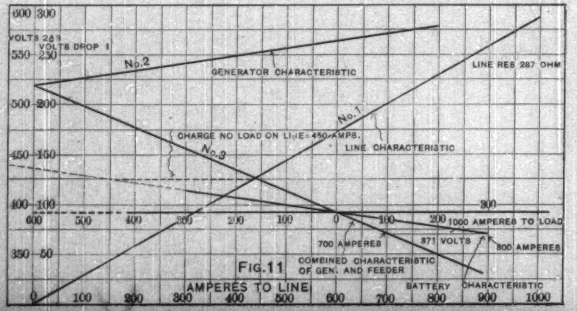



FIG. 11

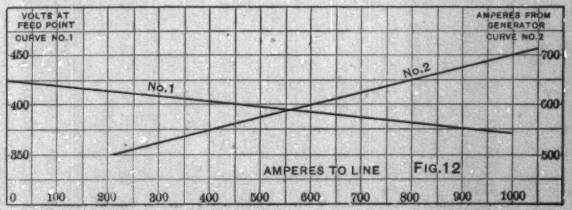



FIG. 12

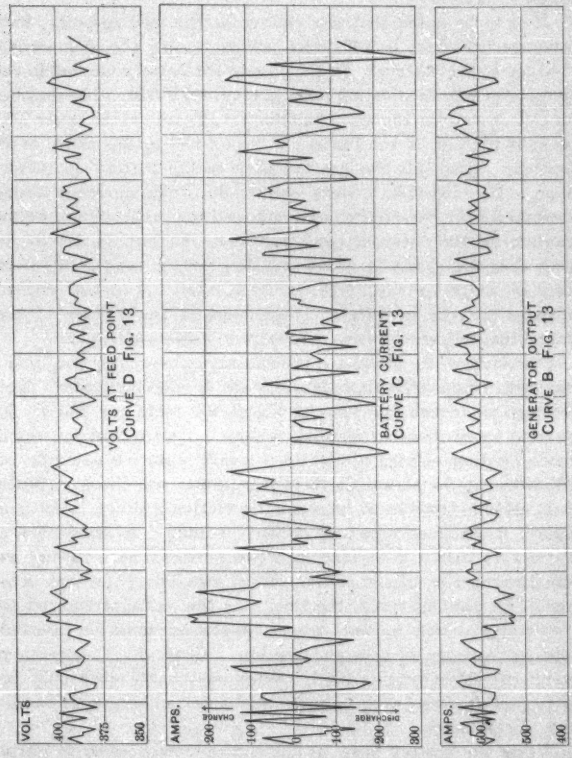
it not only performs all the functions of equalization with the production of the accompanying benefits, but the copper and size of generator are so much decreased that the saving is greater than the cost of the battery, so that the first cost of the system is less than in the case of a generator without the battery auxiliary.

It is to be noted that the voltage at the feeding-point, with average line-load, is 392 volts, which is also the open-circuit voltage of the battery. In the case of the battery located in the power-station, the floating voltage required is 550. The number of cells in series in the former instance is 194, and in the latter 271; so that the cost of the battery is diminished in this ratio, or 28 percent. Also this changes the slope of the battery characteristic. The 550-volt battery shows 30 volts increase above normal for the one-hour charge rate, and the same amount below normal for the one-hour discharge rate. In Fig. 11 it may be seen that the variation above or below normal for the one-hour rate of charge or discharge, is, 21.5 volts due to the smaller number of cells in series. The change in the voltage of the individual cells is, of course, the same in either case.

*Overcharge.*—In practice it is necessary to overcharge or "boil" the cells occasionally to maintain them in good condition. The voltage on overcharge rises to about 2.6 volts per cell or 30 percent more than the normal voltage. Thus a battery which has a floating voltage of 550 volts would require a potential of 715 volts at the terminals to give a proper overcharge. Since it is seldom possible to increase the station voltage to such a figure, it becomes necessary when "boiling" to connect the battery so that it is divided into two portions, each having an equal number of cells in series. These two halves are then connected in parallel across the line. As the voltage required to overcharge is now halved, proper resistances must be inserted between each set of cells and the line. Generally, the ordinary water-barrel rheostat is used. Since overcharge need only be given ten or twelve times a year, this is not a continuous condition of operation.

Where the battery floats at the end of a long feeder of fairly high resistance, which is heavily loaded, the normal battery voltage is lower than that of the station, and during the early hours of the morning, when for half or three quarters of an hour the load may be thrown off the feeder, the decrease in feeder-drop, together with full generator excitation, may bring the

voltage at the battery terminals up sufficiently high to give it the required overcharge. In the case just cited, the voltage at the feed-point with 75 amperes passing over the line (which current is ample for boiling a 600-ampere-hour battery) is 502




volts. The potential required at the battery is $194 \times 2.6 = 503$ volts. A very slight increase in the generator voltage will, therefore, meet the requirement, if the feeder be entirely relieved of other load.

### Comparison Cases I. and II.

The results, then, of locating a battery out on the line as compared with the use of the generator working alone are:

(1) Load on generator is maintained much more constant as indicated in Curve B, Fig. 13.

(2) Copper cost is greatly reduced.

(3) Voltage varies much less, and with less copper the minimum voltage is increased.

Also the full capacity of the battery is utilized, which was not the case when floated in the station.

### Booster Systems.

In discussing the operation of the two batteries previously considered with booster auxiliaries, it is proper here to indicate how, and under what conditions, automatic boosters operate. There are two generic types of regulating boosters: One in which slight changes of line voltage cause changes in field excitation to produce charge or discharge; in the other the field excitation is varied by the changes in external load.

Fig. 14 shows an example of the first-mentioned type: G is a shunt-wound generator, B the booster armature, E the battery, and M M the motors or variable load on the system. The driving-motor is not shown. The operation of this booster is as follows: At normal load, the voltage of the battery and line are equal, and as no current flows either into or out of the former, the series field, S, is not excited and the booster voltage is zero. On increase of external load, the generator tends to send out additional current, causing a slight decrease in line voltage. The battery then begins to discharge and the field, S, being energized by the outgoing current, induces an e.m.f. to assist the discharge, or in other words, adds an e.m.f. to the battery e.m.f., equal to the drop due to discharge. This added e.m.f. compensates for the battery-drop whatever the rate of discharge, since both vary directly as the outflowing current. Conversely, if the load decreases, the line voltage will rise slightly, causing the beginning of a charge. The direction of the current now being reversed in the coils, the e.m.f. produced in the booster armature is also in the opposite direction and assists in forcing the charge into the battery. Since the rise in battery voltage on charge and the booster e.m.f., both increase proportionally to the current, the rise in battery e.m.f. is practically compensated for. Therefore, within small percentages the voltage is maintained constant, regardless of the rate of battery charge or discharge,

and from this it follows that the load on the generator is also kept constant. Obviously, this booster is operative only on systems having a falling characteristic with load increase; that is, with a shunt generator, if the battery is located in the power-house or on a feeder, the drop through which exceeds the increase in voltage at the generator terminals, if the system is fed by a compound-wound dynamo.

The second type of booster is shown in Fig. 15. In this the battery voltage is equal to that of the line, and at normal load the excitation of the series field s, which is in the external circuit, is exactly equal to that of the shunt field *f*. These being oppositely wound, neutralize, and the booster voltage is zero. If the external load be increased, the excitation of s overpowers that of *f* and there results a booster e.m.f. proportional to the increase in external load and in a direction to cause the battery to discharge.

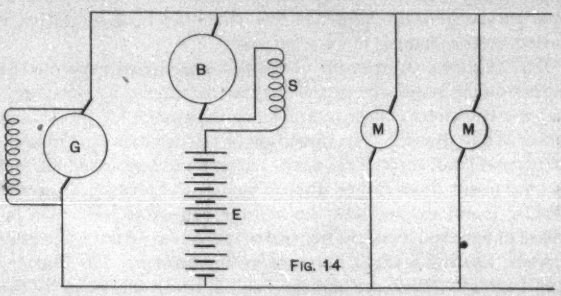



FIG. 14

Should the load decrease, the shunt field is the predominant one and produces an e.m.f. in the booster armature to cause a charging current to flow into the battery. This booster can be used either with shunt or compound-wound machines.

These diagrams merely illustrate the salient principles of booster operation, and are not complete practical devices, as additional windings are required to produce points of stable equilibrium and prevent excessive charge or discharge. There is a large number of different types having various windings and connected in ways peculiar to the ideas of the inventors, or the limiting factors of existing patents. These two, however, show the principles on which they all work.

CASE III.—BATTERY WITH BOOSTER IN POWER-STATION.

Since with a booster-controlled battery, the generator load is kept

practically constant, it matters little what kind of generator is used in the power-station, it being only necessary to settle on the best average station voltage. Assuming 550 volts as the desired e.m.f. at the station, a 550-volt battery, the same load and general conditions as in Case I, the size of battery is to be determined. Since the average load is 600 amperes, and the maximum 1,000, a battery to discharge 400 amperes is required. If the one-hour rate is not to be exceeded, an 800-ampere-hour battery is necessary. The maximum load, however, comes on the system but infrequently, as shown by the load-diagram, and as it lasts only 5 seconds, a 700-ampere-hour battery, having a one-hour rate of 350 amperes, will be ample as it will only have to give out 17 percent in excess of its rating, and then for a negligible time. Even a 600-ampere-hour battery would work satisfactorily on this load.

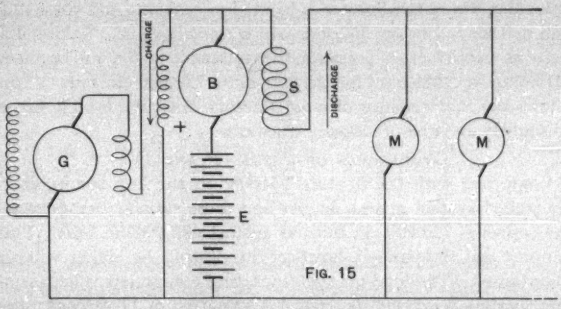



FIG. 15

Assuming a 700-ampere-hour battery in the station, practically all the fluctuations in load will be absorbed and the generator load-curve becomes a straight, or rather a wavy line. The station voltage being constant, the feed-point e.m.f's at various line-loads are not quite so satisfactory as are furnished by a compound-wound generator. If the minimum voltage at the feed-point is to be 350, the resistance must not exceed .2 ohm or .1418 ohm for the copper resistance, requiring 805,000 cir. mils. as against 665,000. The generating equipment, however, is reduced in size from 550 kw to 350 kw and the fuel economy greatly increased.

### COMPARISON OF CASES I. AND III.

As compared with the shunt-wound generator working with a floating battery in the power-station, the generating equipment

is 330 kw as against 440; the copper, 805,000 circular mils as against 945,000; the load variation 6 percent as against 47 percent; the voltage variation at the feed-point 19 percent as against 20 percent—these two factors being practically equal, because of the greater amount of copper in the line fed from the floating battery.

CASE IV.—BATTERY AND BOOSTER OUT ON LINE.

If the floating battery out on the line has a booster auxiliary working with it, the current passing over the feeder will be maintained constant, and the average load only will be transmitted from the power-station. If, as in the previous cases, the voltage is not to fall below 350 volts, the resistance (assuming 550 as the generator voltage) will be .333 ohm total or .2751 ohm for the copper; equivalent to 415,000 cir. mils. This, however, is not so advantageous as it seems, for the voltage at the point where the booster is located is constant and the e.m.f. will neither fall below 350, nor will it rise above it. Taking 420 volts as a satisfactory pressure, the resistance of the line becomes .217 ohm = .1588 ohm for the copper = 720,000 cir. mils. This allows the maintenance of a satisfactory potential which varies so slightly as to be practically constant.

COMPARISON OF CASES II. AND IV.

Compared with the floating battery on the line, the booster-controlled system gives a higher and more satisfactory voltage; the copper is 720,000 cir. mils. as against 495,000 cir. mils. This is not a real comparison, however, in view of the higher voltage maintained. The load on the generating equipment is kept constant within 6 percent as against a variation of 17 percent; the power-house capacity required is 330 kw against 385 kw.

While the booster-controlled battery out on the line is theoretically superior to a similar equipment in the power-station it requires extra attendance, which in many cases offsets its advantages and makes it advisable to put the booster and battery in the station and add a little more copper to the line, or install a floating battery at the feed-point.

Furthermore, the usual condition in practice is that of a power-plant supplying current to numerous feeders, no single one of which carries sufficient current, or is enough longer than the others to warrant a battery installation only to take care of its load and voltage. Generally, therefore, in order to regulate the station load, the battery and booster must be in the power-station.

The advantages shown by the booster-controlled batteries over those floating across the circuit, do not include a most important one; namely, the ability to obtain full battery discharge even when the stored energy in the cells is nearly exhausted and the voltage decreased below normal. The battery characteristic (Fig. 1) shows that the minimum voltage, when discharging at the hour rate, is 1.915 volts per cell. This is the case only when the battery is worked nearly fully charged, and the output and input are so nearly equal, that the state of battery charge is practically unchanged. If, however, any unusual condition or event should raise the average load on the system, the output might exceed the input for several hours, and the battery becomes so far exhausted that the voltage would fall to 1.7 per cell when discharging at the one-hour rate. This is a decrease of .215 volts per cell—or 58 volts in a 550-volt battery—below the minimum voltage of discharge when the battery is worked near the point of full charge. Obviously, without a booster the battery in this condition would not be of any assistance at all in the power-station, and its regulating ability out on the line greatly impaired.

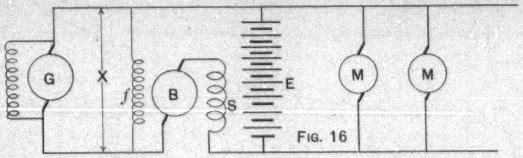


FIG. 16

Another advantage offered by the booster is, that the necessary overcharge is not attended by a troublesome manipulation of water-barrels, with the usual accompaniment of heavy grate-bars and a jug of sulphuric acid.

CASE V.—BOOSTER IN STATION; BATTERY ON LINE.

The final method of installing the battery, that is, with the booster in the power-house and the battery out on the line, is peculiar, in that only one type of booster may be used, and there must be a separate feeder from the booster to the point where the battery is located. In the foregoing discussions, a single feeder from the power-station to the feed-point has been assumed, to simplify the comparison, which is just as valid in its results as if the load had been distributed over the length of various feeders. If the circuits in the cases discussed were merely a simple trolley and track with distributed loads, any of the four

described applications could be used, except that the booster system shown in Fig. 15 would not be suitable as the auxiliary for the battery floating on the line.

*Character of Booster.*—A particular arrangement of the type of booster shown in Fig. 15, causes it to allow only a practically unvarying current to pass through its armature. Referring to Fig. 16, G is the generator; B the booster armature; *f*, a shunt field; S a series field; E the battery, and M M the motors or variable load. The voltage across the mains at X is obviously the generator voltage, while the potential at the terminals of M M is equal to the voltage at X plus that of the booster. As in the case of the booster shown in Fig. 15, the shunt and series fields are in opposition; but at normal load, the magnetization of the shunt field exceeds that of the series field, and an e.m.f. is generated in the booster armature, which is added to that of the generator to

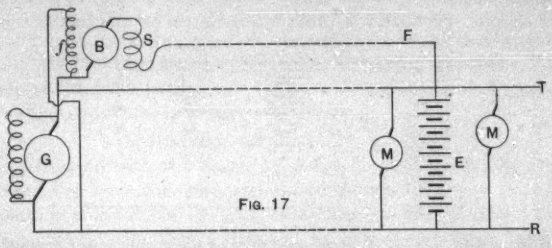



FIG. 17

produce normal voltage across the mains. The battery being connected directly across the mains, its normal voltage is equal to that of the generator, plus booster.

If an excess load should come on the system, the generator tends to send additional current to the line, strengthening S and reducing the booster voltage and, consequently, the potential across the mains. The battery then discharges. The converse occurs on increase of load; the booster voltage rises, and current flows into the battery. This system allows only a constant current to pass through the booster armature and is therefore known as the constant-current booster.

In Fig. 17 is shown the method of applying this machine to the case in hand. The symbols are the same as in Fig. 16; T is the trolley, R the track and F the feeder from the booster. The shunt coil *f* is connected across the line and coil S in series with the feeder F.

*Line-Drop.*

In Fig 18. the curve A represents the voltage which obtains along a line fed from one end, and with the load uniformly distributed along its length.

In the figure, the drop at the distance L, from the source of is seen to be 350 volts. If from P only one half the circuit is supplied with current, there will be a decrease in drop due to lowered resistance, and a further decrease due to the fact that only one half the current flows over the feeder. If the distributed load from P to E is 1,000 amperes and the drop 350 volts, the load over one-half the distance is 500 amperes and the resistance of the circuit one half that from P to E, so that the drop is one fourth of 350 or 87.5 volts, if the circuit extend only to N. This shows that the drop at the end of a main supplying a distributed load varies directly as the square of the distance over which it feeds.

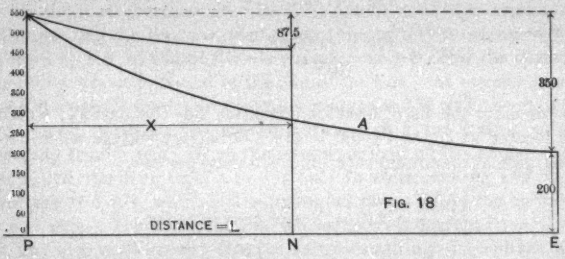



FIG. 18

*Data of System.*

Assume the same conditions as in previous cases and shown in Fig. 19, taking the load as being equally distributed along the line, which in this case, however, is 5 miles in length instead of 2 miles, as before. Computations for drop on a 2-mile line with this system show the cross-section of the copper to be so small that it has not the requisite current-carrying capacity. The copper which extends along the entire length of the circuit will be taken as 400,000 cir. mils. inclusive of the trolley wire. The resistance per mile of circuit is .1424 ohm for the copper and .0291 for the track, making .1715 ohm per mile, or .8575 ohm total. The drop on distributed load is 429 volts at the far end of the line when 1,000 amperes pass over it. This is shown by Curve No. 1 in Fig. 19. It is desired so to place the battery on

the line as to reduce the drop to 150 volts, putting the booster in the power-station, and adding such an amount of copper as may be necessary.

*Location and Voltage of the Battery.*

Since the drop from the power-station varies as the square of the distance over which a distributed load is supplied, the distance, $X$, to which the original copper can feed at a drop of 150 volts, is computed by the proportion:

$$X^2 : (5)^2 :: 150 : 429, \text{ from which}$$

$$X = 2.89 \text{ miles, which is shown at point B.}$$

The remaining distance, or 2.11 miles, is to be divided into two equal parts and the battery so located that it will feed in each direction 1,055 miles, which is its distance from the end of the line. Point C. shows its position. Its voltage is fixed by the condition that the line voltage must not fall below 400 volts. Applying the above proportion:

$$(5)^2 : 429 :: (1.055)^2 : Y$$

$$Y = \frac{429 \times (1.055)^2}{(5)^2} = 19.1 \text{ volts}$$

= drop for the distributed load on either side the battery, which is located at C. $19.1 + 400 = 419.1$ = battery voltage when discharging at maximum rate, or, 1.915 volts per cell.

$$\frac{419.1}{1.915} = 219 = \text{No. of cells in series.}$$

$219 \times 2.025 = 443.47$ = voltage at C, with normal load on system, battery neither charging nor discharging, 2.025 being the open-circuit voltage per cell as shown in the characteristic (Fig. 1). Voltage per cell is 2.14 when charging at maximum rate.

$2.14 \times 219 = 468$ = volts at battery at maximum charge.

*Feeder to Battery.*

The total current to that portion of the system past B away from A, must be supplied by the feeder F (Fig. 17), and this current $Z$ is determined by the proportion.

$$(5 - 2.89) : Z :: 5 : 600, \text{ whence}$$

$$Z = 253.2 \text{ amperes.}$$

The size of the feeder F is now to be decided on. If its current-carrying capacity be sufficient to allow 253 amperes to flow, its

resistance is of little moment so far as the operation of the system is concerned, since the voltage of the booster can be made sufficiently high to compensate for any feeder-drop. This resistance, however, is commercially a serious factor, since the booster energy input is lost. The question, then, is to determine the most economical cross-section of copper, which is beyond the scope of this paper. For purposes of the discussion assume it to be 400,000 cir. mils.; resistance .1424 ohm per mile. Resistance of circuit = .1424 + .0291 = .1715 ohm per mile; or .1715 × 3.95 = .6774 ohm total. The drop then to c = .6774 × 253 = 171.4 volts 443 + 171.4 = 614.4 = voltage of generator plus booster at the station 614.4 − 550 = 64.4 = booster voltage when average load is on the line. Since the feeder-current is constant, the drop to C is also constant, hence for variation of battery voltage to cause charge or discharge, the booster voltage must vary as indicated at A. The straight lines 10, 11, 12 show the drop along the feeder.

*Changes with Load Variations.*

As the load decreases and the voltage along the line rises, the distance that the line will feed toward the battery is increased, as shown by curves 4 and 6, while curves 5 and 7 show the corresponding rise of potential at the end of the line. At normal load of 600 amperes total, the minimum voltage is 438 and the point of lowest potential has moved outward to E, the station now feeding from A to E. When the minimum load of 260 amperes is supplied, the drop from A to C is only 73 volts, making the line voltage at c, 477, which is 5 volts higher than that of the battery on charge. Therefore, the line will supply all current from A to C, and also send enough into the battery to bring the voltage at the line down to 468; which extra current to the battery is 9 amperes.

*Battery Capacity.*

Since the battery regulates 2.11 miles of the system, the average current as shown is 253 amperes, while the maximum is $\frac{1000 \times 2.11}{5}$ = 422 amperes. Therefore, the one-hour discharge rate should be 422—253 = 169 amperes, making the battery capacity 340 ampere-hours. The constant current from the booster, however, must be taken care of. On light load the line is furnishing all current up to C, and also 9 amperes to the battery, making 262 amperes flowing to this point. The current consumption from C to D is only $\frac{260 \times 1.055}{5}$ = 54.8 amperes; 262—

54.8 — 207.2 amperes, going to the battery which should not be in excess of the one-hour rate of charge. This necessitates the use of a 414-ampere-hour battery.

*Results.*

This system then distributes current over five miles with a maximum current of 1000 amperes; the minimum voltage is 400, the average 438; the fluctuations on the system are from 260 to 1000 amperes, on the generator from 458 to 831. This is accomplished by the use of 9 miles of 400,000 cir. mil. feeder, a 24-kw booster, a 450-kw generator, and 261 cells of 400-ampere-hour capacity.

This paper has covered, it is true, only one set of conditions, but from it can be learned the method of investigation applying to any particular set of conditions; and it also shows that the setting forth of any general formulas or deductions to be applied indiscriminately is practically impossible.

## Discussion on Mr. Lyndon's Paper.

J. R. Appleton.—The value of a battery for relieving the strain on the generators with fluctuating loads is well illustrated by a plant at Oakland, California. The station equipment comprises one 600- and one 850-kilowatt generator; without the battery it was essential to keep a man at the throttle of each engine, on account of the rapid variations of load; with the battery, the 850-kilowatt generator maintains the load between 700 and 800 amperes continually, and the battery supplies current up to 2,600 amperes; that is, a constant load is kept on one generator, and the other generator becomes a spare unit.

J. L. Woodbridge.—It may be worth while to point out that the function of a battery at the station is different from that at the end of a feeder. The station battery will keep the station load constant and regulate the station voltage; the feeder battery will not hold the station voltage constant to any material extent, and will not reduce the load fluctuations materially.

W. E. Goldsborough.—The introduction of a storage-battery at a center of distribution of the St. Louis Rapid Transit Company's system has made a saving in copper which pays for the installation of the battery and all the auxiliary machinery used in connection with it.

J. W. Lieb.—Though not immediately germane to the paper, it may be worth while to point out the very great importance of a storage-battery in a large alternating current generating station to supply field excitation. It is imperative that batteries should be installed for this purpose which may be thrown instantly on the field circuit to take the place of the other exciters; for this purpose, the battery capacity should be large. The speaker has made a test on a 4000-kw. plant, throwing out the motor-generator exciters and throwing the whole exciter equipment on the battery; there were practically no fluctuations in the bus-bar voltage. In the speaker's opinion, it is also imperative that the storage-battery should be used as a reserve in any general direct-current distribution system supplying a large population; the possible causes of interruption to service are so numerous, that such a system cannot be considered safe without a very large reserve in battery.

W. W. Donaldson.—A battery on a line should be installed so that the greatest possible amount of work will be done by the battery; if it discharges only once a day, it is not doing as much work as it should. This question of amount of use during the day should be given consideration.

W. E. Goldsborough.—A battery should unquestionably be adjusted to give the maximum return to the equipment with which it is connected. A battery supplies a means for keeping all the apparatus working at the point of maximum efficiency, from the central station, through the sub-stations, and on to the line. In a large system, the small loss of energy in the battery is unimportant in comparison with the reduction of depreciation which it brings about in all other parts of the system.

A. S. HUBBARD.—In a railway power-station, the average load is not the same at different periods of the day A booster should be capable of adjustment to change the average load supplied by the generator. If a battery is proportioned for the daily average, it will be far larger than necessary to care for the momentary peaks; by proportioning the booster so that the average load, as represented by the generator load, can be raised or lowered, as conditions demand, the size of the battery may be reduced. A similar consideration applies to the floating battery when the car schedule is changed; but on a system using a constant-current booster in a power-station with a separate feeder to the battery, a change of schedule can be cared for by changing the booster output. Further, if a battery is operated on a daily average, its efficiency is far less than if operated for comparatively short periods—say, of two or three hours.

LAMAR LYNDON.—The various points presented by the different speakers, while interesting, do not seem to be directly germane to the paper in hand. There is therefore nothing to be added to the statements of the paper.

# NOTICE.

This number of the TRANSACTIONS is delayed in distribution by a fire which destroyed our print-shop on the night of December 28th.

The first five forms of this number had been printed and delivered to the binder on December 21st; the last two forms were on the presses when the fire occured and were totally destroyed.

This fire also destroyed a part of the electrotype plates of Volumes XX (September to December 1902) and XXI (January to June 1903). In addition to these losses there is the further and complete loss in type of all the pages of Volume XXII (July to December 1903,) which must be reset. The December issue is to contain the revised catalogue which was partially destroyed.

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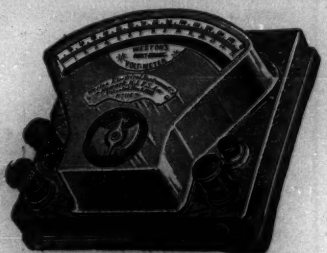

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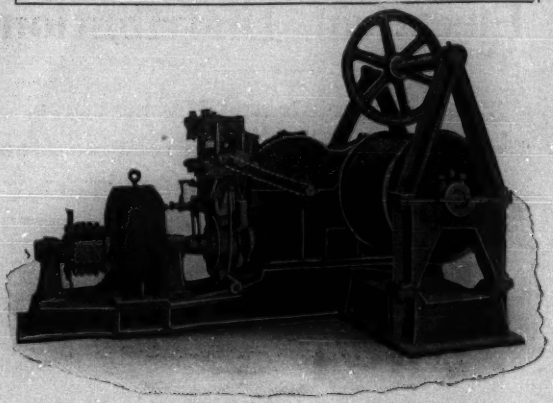

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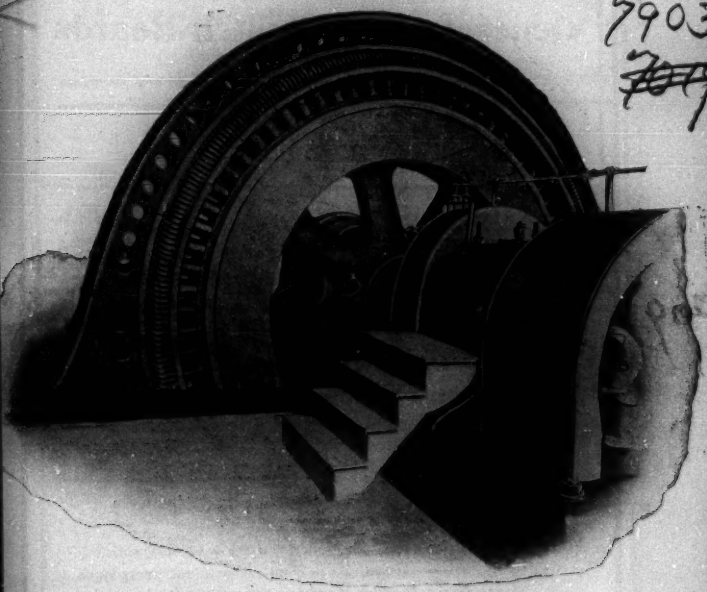

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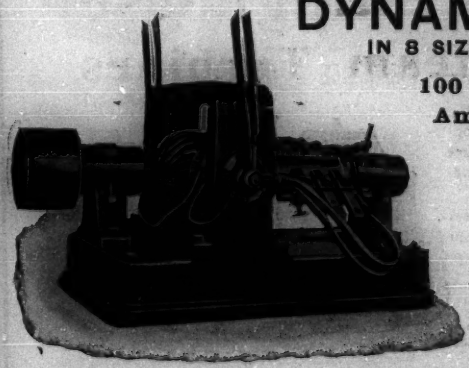

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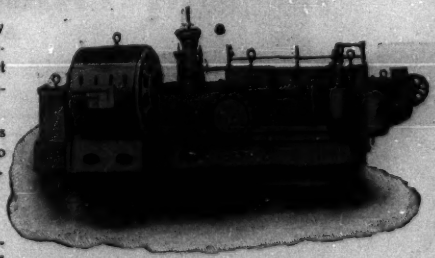

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